Re-evaluating Okun's Law: Why all recessions and recoveries are "different"

Sacha Gelfer*

June 2020

Abstract

This paper explores the relationship between GDP growth, the unemployment rate and employment growth. Using a structural DSGE model and a data-rich estimation approach I am able to estimate the coefficients and correlations between GDP growth and unemployment rate changes, GDP growth and overall employment growth as well as GDP growth and employment growth by sector. I find historically equivalent estimates when I compare the simulated model with actual realized data. I am then able to look at the effect different types of economic shocks have on these estimates and I find that investment and finance shocks have larger effects on employment growth and the unemployment rate when compared to productivity and other supply-side shocks when the effect on GDP is controlled for. For example, the model suggests that a 1% decline in real GDP would result in an increase of the unemployment rate of 0.5% if it was caused by a financial or investment shock, however, it would only increase the unemployment rate by 0.15% if the 1% decline in real GDP was caused by a productivity shock.

Keywords: Okun's Law, DSGE-DFM, Financial Recessions, Data-rich DSGE
JEL: E17, E24, E32

*Bentley University, Waltham, MA, USA. E-mail address: sgelfer@bentley.edu

1 Introduction

In one of the few described "laws" in economics, Okun (1962) revealed a negative relationship between unemployment and real output. Since its finding Okun's law is a bedrock of macroeconomic theory and a key component of business cycle examination. This paper will focus on the following two specifications of Okun's law

$$\Delta U_t = \alpha + \beta \Delta Y_t \tag{1}$$

$$\Delta E_t = \alpha_e + \beta_e \Delta Y_t \tag{2}$$

in which ΔU_t is the change in the unemployment rate, ΔY_t is the percentage change in real output and ΔE_t is the percentage change in overall employment.

Recent literature has focused on examining Okun's law over the last few business cycles. Gordon (2010), Owyang et al. (2012) and Grant (2018) all find instability in the parameter estimates of the relationship between the unemployment gap and the output gap. Owyang et al. and Grant find a significant increase in absolute terms of Okun's slope coefficient (β) over the Great Recession and its recovery. While Ball et al. (2017) and Conraria et al. (2020) find that β is historically stable over the past 70 years with only small fluctuations around given time periods.

This paper is able to bridge these key findings and explain why we have seen short-run variations in Okun's slope coefficient in times often associated with so called "slow" or jobless recoveries while still observing a stable Okun's slope coefficient in the long-run. The model I use to conduct my analysis is a dynamic stochastic general equilibrium (DSGE) model with financial frictions developed by Del Negro et al. (2013). I use the technique of Boivin and Giannoni (2006) and Gelfer (2019) and estimate the model in a data-rich environment using a large data vector of macroeconomic time series (DSGE-DFM). This allows an avenue through which DSGE environments can be used to study such series as the unemployment rate, unemployment duration and employees by sector even when no such series are directly incorporated into the structural model. Instead, the series are allowed to load on economic variables and structural processes that are inside the DSGE model. This allows me to examine the dynamics of data series important to the goal of this paper while still keeping the structural foundation of the model.

I find evidence that financial and investment shocks are associated with higher levels of unemployment and longer average unemployment duration in comparison to responses to other types of shocks with identical output decreases. I also find that sectors associated with more capital intensive operations (manufacturing and construction sectors) are the very sectors that are slowest to recover from a financial shock. These results suggest that the relationship between unemployment and GDP growth implied by Okun's Law depends on what mechanism is behind the output change.

To further examine this state-dependent Okun's law relationship, I simulate the DSGE-DFM model when all structural shocks in the DSGE model are active and estimate the implied Okun's law slope coefficients on the simulated data. I find that the estimates of β and β_e in equations 1 and 2 are similar for the simulated data and the actual realized data. I then simulate the DSGE-DFM model eight additional times, allowing only one structural shock to be active at a time. This allows me to estimate Okun's law slope coefficients that are associated with each structural shock. I find that Okun's law slope coefficients are quite different depending on what is causing the change in real output.

The slope coefficients are largest in absolute terms when changes in output occur because of investment or wage shocks and smallest when the change in output is caused by consumption or productivity shocks. This is true if the unemployment rate or employment level is on the left side of Okun's law. Examining Okun's law when sectoral employment is on the left side, I find that investment shocks and financial shocks cause the greatest change to manufacturing and construction employment, while wage shocks cause the biggest change to service employment. In all, the analysis implies an historically stable Okun's law slope coefficient in the long-run, but significant time variation in the coefficient if one type of structural shock is more dominant over a short-run time frame.

The results of the paper are consistent with other empirical and theoretical work that suggests that sluggish labor market recoveries may be directly linked to what initiated the preceding recession. In particular, Boeri et al. (2013) found that firms in industries that use more temporary financing in every-day business operations adjust employment levels much more when credit shocks decrease liquidity than firms with less financing on their balance sheet. This liquidity channel leads to larger job loses and slower hiring when a decrease in economic output is caused by a financial shock. A result that is also found by Chodorow-Reich (2013) and Duygan-Bump et al. (2015). Further, Schmitt-Grohe et al. (2017) find that negative investment and financial shocks that push the economy to the zero lower bound slow down recoveries and that employment growth can remain low even as productivity and output growth returns to their long-run levels.

The remainder of this paper is structured as follows. Section 2 gives a brief overview of the model and estimation routine used to produce my results. Section 3 presents impulse response functions (IRF's) for unemployment, overall employment and sectoral employment for different "types" of normalized output declines induced by the various structural shocks inside the DSGE-DFM model. Section 4 examines the estimated Okun's law relationship implied by the simulated DSGE-DFM model. Section 5 concludes and discusses future extensions.

2 The Structural DSGE Model and Estimation

In this paper, I consider a DSGE model based on the FRBNY model outlined by Del Negro et al. (2013) and discussed in detail by Gelfer (2019). This model is an extension of the Smets and Wouters (2003, 2007) New Keynesian model with the addition of a credit market with frictions that closely follows the financial accelerator model created by Bernanke, Gertler and Gilchrist (1999). The model involves a number of exogenous shocks, economic agents, and market frictions. The agents include households, intermediate and wholesale firms, banks, entrepreneurs, capital producers, employment agencies, and government agencies.

The exogenous shocks of the DSGE model will be central in the analysis of this paper. In all, the model has eight exogenous shocks, two policy shocks, a financial shock, three supply shocks and two component demand shocks, all of which are described in Table 1.

I estimate the model using specific data that matches particular states and data that have no direct connection to the model's endogenous variables, I call this method DSGE-DFM estimation. In the remaining subsection, I give an overview of the DSGE-DFM estimation method.

Table 1: Structural Shocks of the DGSE Model

Shock	Description	Initial Agent Exposure
Policy Shocks		
Monetary Policy Shock	An unexpected shock to risk free interest rate	Monetary Authority
Gov't Policy Shock	Spending shock to the Gov't portion of GDP	Fiscal Authority
Financial Shock		
Finance Shock	Risk shock that decrease or increase the spread between the bank deposit rate and the bank lending rate	Entrepreneurs & Banks
Supply Side Shocks		
Productivity Shock	Shock to productivity affects firm production	Intermediate Firms
Price Shock	Shock to the mark-up above marginal costs monopolistically competitive firms charge final good producing firms	Intermediate Firms
Wage Shock	Shock to the monopolistic power households have over their specialized labor	Households
Demand Shocks		
Preference Shock	Shock to the discount rate that alters households consumer and savings decisions	Households
Investment Shock	Shock that affect the marginal efficiency of investment as in Justiniano et al. (2011)	Capital Producers

2.1 DSGE-DFM Estimation

Bayesian estimation of a DSGE model in a data-rich environment incorporates a state space model set-up and is characterized by equations (3)-(5).

$$S_t = G(\theta)S_{t-1} + H(\theta)v_t \text{ where } v_t \sim NID(0, I_m) \tag{3}$$

$$X_t = \Lambda S_t + e_t \tag{4}$$

$$e_t = \Psi e_{t-1} + \epsilon_t \text{ where } \epsilon_t \sim NID(0, R) \tag{5}$$

Here e_t follows an AR(1) process and is often referred to as measurement error. The matrix X is J x T where J is the number of data series used in estimation and T is the number of observables for each series. Unlike in traditional DSGE estimation the Matrix Λ is now no longer assumed to be known by the econometrician, but instead is estimated within the MCMC routine. The matrices Ψ and R that govern the measurement error's stationary processes for each series are assumed to be diagonal and are also estimated within the routine.

The measurement equation (4) has the following structure:

$$
\begin{bmatrix} Output\,\#1 \\ Output\,\#2 \\ Inflation\,\#1 \\ Inflation\,\#2 \\ \vdots \\ ------ \\ [Expendture\ Components] \\ [Labor\ Market] \\ [Yields] \\ [Credit\ Market] \\ [Price/Wage\ Indexes] \end{bmatrix} = \begin{bmatrix} 1 & 0 & ... & 0 \\ \lambda_{Y1} & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ 0 & \lambda_{\pi 1} & ... & 0 \\ -- & ---- & --- & -- \\ [\lambda_{E_1}] & [\lambda_{E_2}] & ... & [\lambda_{E_n}] \\ [\lambda_{L_1}] & [\lambda_{L_2}] & ... & [\lambda_{L_n}] \\ \vdots & \vdots & ... & \vdots \end{bmatrix} \begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \\ \vdots \\ \epsilon_t^f \end{bmatrix} + \begin{bmatrix} e_{t,1} \\ e_{t,2} \\ \vdots \\ e_{t,J} \end{bmatrix}
$$

where X_t is partitioned into core series and non-core series separated by the dashed line. The core series are series that are only allowed to load on one particular variable of the state vector, S_t, to which there is a known sole relationship between series and state. (For instance, GDP to Y) Further, the factor loading coefficient for the first series of each core variable that corresponds to a particular known state is assumed to be perfectly tight, this is represented by the 1's in the Λ matrix. This anchors the estimated states of the DSGE model and ensures that they don't drift too far away from their theoretical foundation.

The non-core series consists of the remaining data sets not in the core series and are grouped into subgroups. These series are allowed to "load" on all time t states in the state vector. Non-core series may have up to n (where n is the number of elements in S_t) non-zero elements for their corresponding row in Λ unlike the core series whose corresponding row in Λ may only have one non-zero element.

2.2 Data and Parameter Priors

To estimate the model of this paper in a data-rich environment, a total of 97 quarterly data series are used. These series cover the time period of 1984Q2 to 2019Q2. The complete set of series is described in detail by Gelfer (2019). The DSGE-DFM model estimation consist of 17 core series and 80 non-core series. The core series include three measures each of GDP, inflation, employment and nominal interest rates. Also included in the core series are real consumption and investment expenditures and hourly wages and two measures of the interest rate spread.

The series that hold a perfectly tight loading factor are the eight series used in regular DSGE estimation of the model. These include real per capita GDP, the GDP price deflator, per capita real consumption and private investment expenditures, real average hourly wage, hours worked, the annualized federal funds rate and the quarterly spread between BAA corporate bond yields to the 10 year Treasury bond yield. All per capita variables are calculated using the adult population of 16 years and older. These series are either demeaned, linearly detrended log level or log first differenced and demeaned.

The non-core series are grouped into eight categories. In this paper, I will pay particular attention to the *Labor Market* category which includes series that report the overall employment level in the industries of construction, manufacturing, service-providing, wholesale trade, retail trade, financial activities, professional business services, education and health services, leisure and hospitality services and the government sector. In addition, the category also includes a measure of the unemployment rate and the measure of the average duration of unemployment. These series are either demeaned or linearly detrended log level and as is common in the Dynamic Factor Model literature, all non-core series sample standard deviation is normalized to 1.

Finally, the structural parameter marginal priors are in accordance to the Smets and Wouters (2003, 2007) and Del Negro et al. (2013) priors and are given in Gelfer (2019).

3 Comparing the Economic Effects of Normalized Structural Shocks

The DSGE-DFM framework can help in answering questions like: what makes financial recessions and subsequent recoveries so much different than other recessions and recoveries? I attempt to evaluate such a question by comparing the IRF's of different normalized structural shocks. In order to trace the dynamic effects of the structural shocks to additional data indicators I must normalize the structural shocks to assure that output falls by a similar magnitude across the menu of structural shocks in Table 1

To conduct this application, I calibrate all parameters including the loading coefficients of the DSGE-DFM model to their estimated posterior median and normalize the size of the eight structural shocks to ensure that the maximum decrease of real output is equal across the different shocks.[1] This assures any differences in the fluctuations of other variables or series are not due to an output level effect. Figures 1 and 2 examines the IRF's of each structural shock for ten different economic series.

Figure 1 plots the IRF's of real GDP, Investment, Exports and Residential Investment. Notice that by design real GDP decreases by the same amount for each of the structural shocks. However, notice that this decrease in GDP is quickest after negative monetary and consumer shocks, as recovery starts 4-5 quarters after the shock. Recoveries after negative investment and financial shocks start 5-6 quarters after the shock, while recoveries after negative supply shocks (productivity and wage shocks) have more persistence, as they do not begin until 7 or 8 quarters after the initial shock. I also see that particular components of GDP react much differently to what has caused the decline in output. Real Investment and real Exports decrease by a much larger amount and are slower to recover to their steady state value after a financial shock. The decreases in both are similar to that of a negative investment productivity shock but recover at a much slower pace. Recovery for real Residential Investment remains extremely sluggish after a negative financial shock compared to any other type of shock.

When I study the IRF's for different labor market measures, including the unemployment rate and average unemployment duration time, I observe that the effect on both differ depending on what mechanism is behind the output decline. Since the decrease in real GDP is identical, the different unemployment rate dynamics would suggest that the coefficient on Okun's Law is different depending on what the driving force behind the decrease in output is. The unemployment rate increase is largest after negative investment and financial shocks, but the inertia associated with financial shocks is much greater, as the unemployment rate and the average duration of unemployment remains high for much longer when compared to any other type of shock.

[1] The decrease of real output is normalized around the decrease associated with a two standard deviations financial spread shock.

Figure 1: Comparing Normalized IRF's

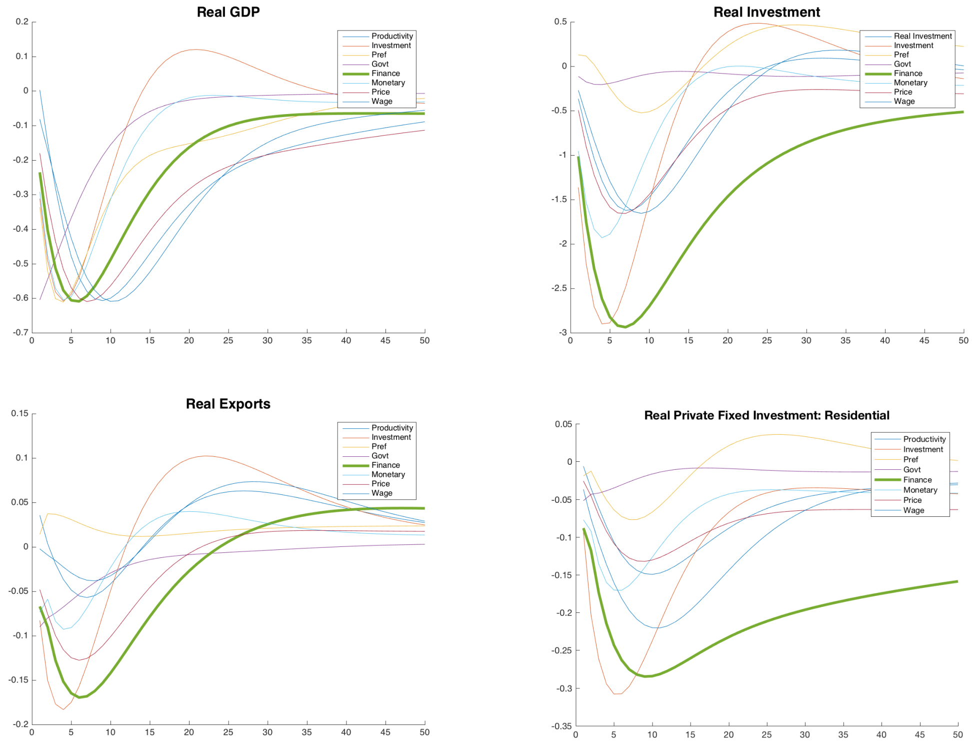
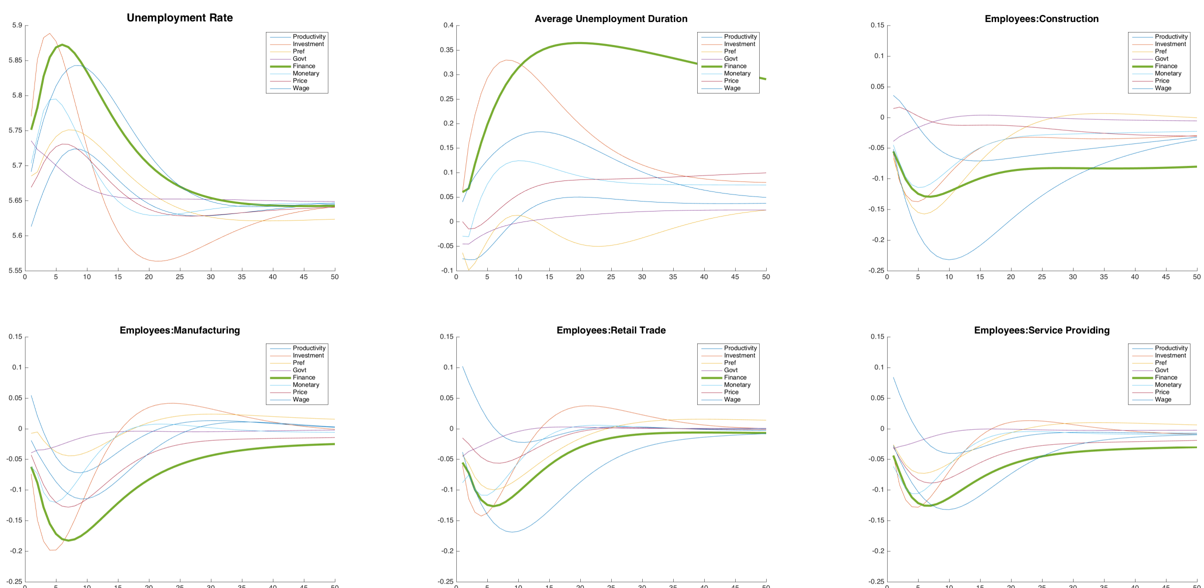


Figure 2: Comparing Normalized IRF's



If I examine the labor market in closer detail it sheds light on why this phenomenon of a high and persistent unemployment rate may occur. I see that the decrease in inventories and real investment are largest and most persistence after a financial shock. As a result the number of employees in manufacturing and construction decreases most significantly after financial shocks, while the decreases of service providing and retail trade jobs after a financial shock are more consistent with those seen after monetary, consumption and investment shocks. This supports the findings of Boeri et al. (2013) as firms in the capital intensive manufacturing and construction sectors rely heaviest on financial markets to operate their businesses.

4 Okun's Law Analysis

In the previous section I saw that the unemployment rate, average unemployment duration time dynamics differ depending on what mechanism is behind the output decline. Since the decrease in real GDP is calibrated to be identical, the different unemployment rate dynamics would suggest that the coefficient on Okun's Law is different depending on what the driving force behind the decrease in output is. To fully examine this concept, I simulate the DSGE-DFM model and estimate equation 1 on the simulated output of the model.[2]

Further, in order to capture the potential different dynamics of Okun's law that the different structural shocks may produce, I simulate the model under nine different specifications. I first simulate the model when all eight structural shocks are active. I then simulate the model an additional eight times, with only one particular shock (described in Table 1) active at a time. The results are plotted and summarized in Figure 3 and Table 2.

I see that the β coefficient on Okun's law for the DSGE-DFM model and the actual data are very similar, estimated to be -0.44 and -0.39 respectively. However, the β coefficient estimates of the model when only one shock is active at a time are quite different. Investment shocks and wage shocks imply a higher β coefficient of -0.5 while consumption, fiscal and productivity shocks imply a much smaller β estimate. These results suggest that the driving structural mechanism behind a recession, recovery or expansion phase will create very different unemployment and output dynamics as illustrated empirically by Owyang et al. (2012), Grant (2018) and Coraria et al. (2020). For example, the model indicates a

[2]Okun's law is estimated using equation 1 with quarterly data where ΔU_t is the % change in the unemployment rate a year ago and ΔY_t is the % change in output from a year ago.

recession driven by large negative investment and financial shocks would result in a large increase in the unemployment rate and recovery driven by positive fiscal and financial shocks would result in a persistent and elevated level of unemployment.[3]

Table 2: Okun's Law Estimates Implied by DSGE-DFM Model

	All	Prod	Inv	Cons	Gov't	Fin	Mont	Price	Wage	Data
Okun's Law Est (β)	-0.441	-0.162	-0.506	-0.082	-0.094	-0.302	-0.275	-0.231	-0.504	-0.386
Std. err.										0.033
R^2	0.580	0.222	0.502	0.082	0.107	0.306	0.397	0.465	0.555	0.492

Figure 3: Okun's Law Implied by DSGE-DFM Model

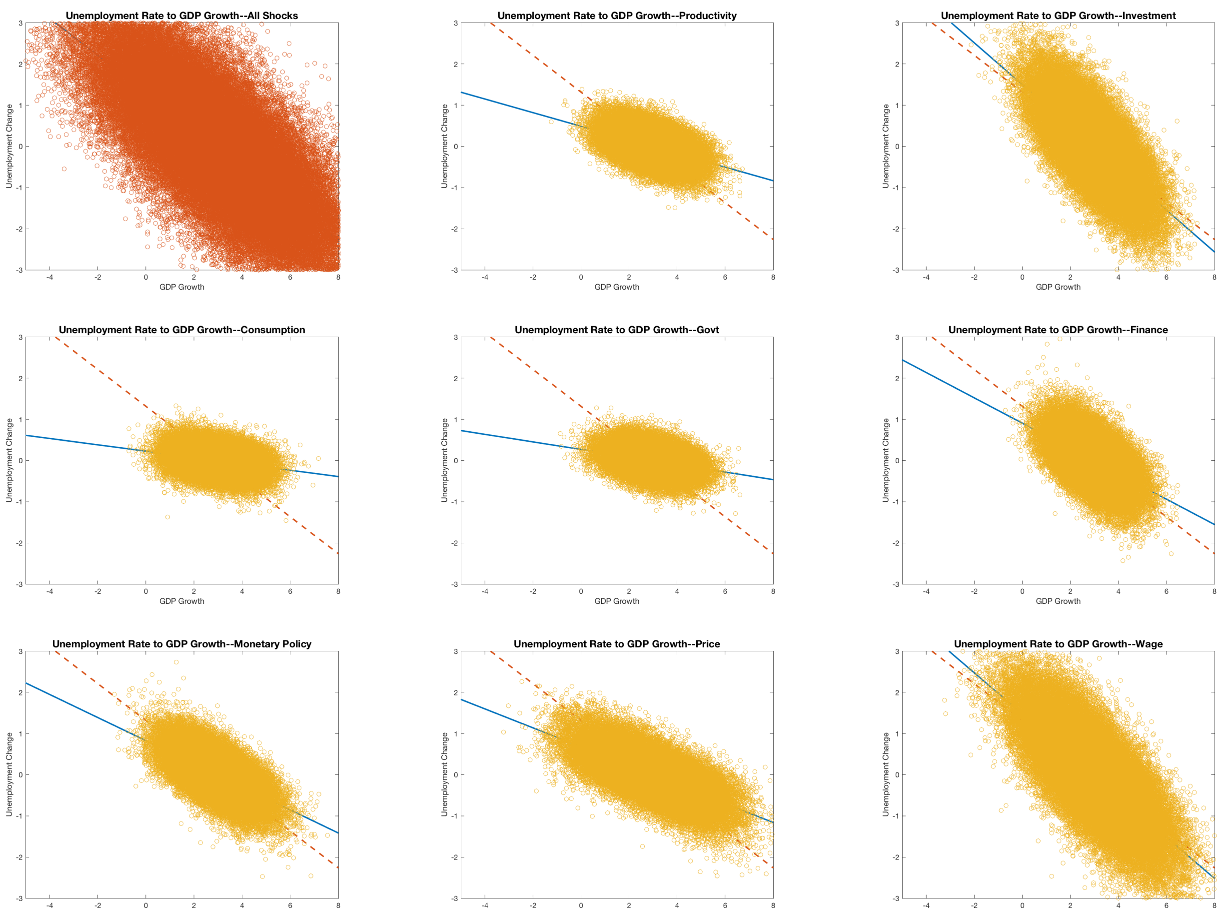


Figure plots the relationship between Unemployment rate change and GDP growth in the DSGE-DFM model, when all shocks are active and when only one specific shock is active in the simulation. The dashed line is a linear regression of Okun's Law when a specified shock is active and the solid blue line is the fitted linear regression line when all shocks are active in the simulation.

[3]The described shocks are found to best describe the Great Recession by Gelfer (2019), Del Negro et al. (2013) ,Christiano et al. (2010).

4.1 Okun's Law for Employment

In addition I examine the less traditional measure of Okun's law that considers the relationship between employment growth and output growth as described in equation 2. A major advantage to the DSGE-DFM estimation technique is that it permits us to consider the economic effects of structural shocks on data series that include overall employment but also employment by sector. The simulated results for all employment and employment by sector are summarized in Table 3 and plotted for all employment in Figure B.1.

Table 3: Okun's Law for Employment Estimates Implied by DSGE-DFM Model

	All	Prod	Inv	Cons	Gov't	Fin	Mont	Price	Wage	Data
All Employees (β_e)	0.628	0.115	0.502	0.327	0.355	0.338	0.412	0.466	0.631	0.744
Std. err.										0.052
R^2	0.722	0.016	0.588	0.341	0.345	0.327	0.461	0.681	0.737	0.602
Manufacturing (β_e)	1.016	0.420	1.138	0.112	0.172	0.628	0.602	0.899	0.864	1.127
Std. err.										0.128
R^2	0.764	0.243	0.623	0.042	0.107	0.339	0.442	0.713	0.694	0.363
Construction (β_e)	1.035	0.245	0.839	0.618	0.154	0.525	0.614	0.016	1.887	2.340
Std. err.										0.186
R^2	0.461	0.047	0.391	0.206	0.025	0.166	0.265	0.000	0.691	0.537
Service Providing (β_e)	0.412	0.136	0.352	0.119	0.062	0.223	0.269	0.303	0.471	0.631
Std. err.										0.045
R^2	0.709	0.098	0.523	0.141	0.056	0.271	0.411	0.611	0.695	0.593
Retail Trade (β_e)	0.509	0.120	0.498	0.178	0.093	0.277	0.325	0.233	0.695	0.872
Std. err.										0.067
R^2	0.609	0.039	0.522	0.127	0.048	0.219	0.328	0.351	0.673	0.550
Financial (β_e)	0.277	0.100	-0.042	0.289	0.023	0.042	0.200	0.084	0.525	0.744
Std. err.										0.081
R^2	0.279	0.023	0.006	0.143	0.001	0.004	0.101	0.037	0.497	0.384
Leisure (β_e)	0.345	0.060	0.426	0.079	-0.027	0.214	0.256	0.192	0.432	0.636
Std. err.										0.062
R^2	0.321	0.005	0.261	0.010	0.001	0.061	0.107	0.109	0.330	0.434
Wholesale Trade (β_e)	0.717	0.230	0.940	-0.034	0.093	0.521	0.432	0.423	0.805	0.857
Std. err.										0.081
R^2	0.635	0.141	0.598	0.004	0.040	0.312	0.375	0.527	0.674	0.450
Government (β_e)	-0.014	-0.044	-0.098	-0.021	0.025	-0.040	0.027	0.036	-0.004	0.166
Std. err.										0.051
R^2	0.003	0.007	0.050	0.002	0.003	0.005	0.003	0.010	0.000	0.072

First, the estimates of β_e for the simulated DSGE-DFM model are consistent with the estimates of β_e for the realized data for all employees. Like Okun's law with unemployment on the left side of the law, the two shocks that create the strongest connection between employment growth and output growth are investment and wage shocks and productivity shocks creating the smallest and weakest connection between the two. When I look at estimating equation 2 with sectoral employment growth on the left side, I see that the largest β_e coefficient both in the data and the simulated DSGE-DFM model are the manufacturing and construction sectors. These are also the two sectors in which the simulated DSGE-DFM model with only investment or financial shocks create the highest β_e estimates. Evidence that aligns with Calvo et al. (2013), Boerri et al. (2013) and the results in the previous section of this paper.

Regarding economic policy, it is worth noting that unexpected monetary policy and fiscal policy shocks that create similar GDP change would also create similar employment growth change for all employees. With the β_e estimated at 0.41 and 0.36. However, monetary policy that generates output growth is much more effective at creating employment growth in the manufacturing and construction sector when compared to output growth generated by fiscal policy. Lastly, we see that the relationship between output growth and employment growth in the services sectors are most effected by wage shocks. As seen by the β_e estimates when only wages shocks were active to be the highest β_e estimates for the service providing sector, retail trade, financial and leisure service sectors.

5 Conclusion

In this paper, I follow the work of Boivin and Giannoni (2006) and Gelfer (2019) and estimate a Dynamic Stochastic General Equilibrium model in a data-rich environment (DSGE-DFM). To explore the economic and labor market effects of various exogenous shocks, I examine structural impulse response functions (IRF's) for series that are usually not obtainable inside DSGE models or only obtainable if embedded in a dynamic factor model with little or no theoretical interpretation of the original shock by which they are generated. However, the DSGE-DFM model creates a structural foundation of what type of initial shock has created the disturbance. I then simulate the DSGE-DFM model and estimate Okun's law coefficients with unemployment, all employment and employment by sector on the left side of Okun's equation.

I see that financial and investment driven recessions have the potential to create prolonged sluggish recoveries and cause the unemployment rate and average duration of unemployment to remain high for a significant time period after the financial shock. A closer look at particular economic series suggests that sectors most likely associated with capital financing (manufacturing and construction) are the sectors that are slowest to recover and sectors less reliant on capital financing (retail trade and service providers) show little to no distinction between financial and investment shocks and other demand and supply shocks.

When the DSGE-DFM is simulated I find that its simulated results give empirically relevant estimates for Okun's law coefficient with regard to unemployment and total unemployment. I also find significant heterogeneity amongst the different structural shocks and the Okun's law coefficient each shock would generate. In particular, investment and financial shocks create the largest Okun's law coefficient while productivity and fiscal policy shocks create the smallest Okun's law coefficient. This suggests that the relationship between labor markets and output would be widely different depending on what the driving mechanism was for the economic contraction or expansion and helps explain the time-varying estimates of Okun's law at certain points over the past 30 years.

References

Aguiar-Conraria, L., Martins, M. M. and Soares, M. J. (2020), 'Okun's law across time and frequencies', *Journal of Economic Dynamics and Control* p. 103897.

Ball, L., Leigh, D. and Loungani, P. (2017), 'Okun's law: Fit at 50?', *Journal of Money, Credit and Banking* **49**(7), 1413–1441.

Bernanke, B. S., Gertler, M. and Gilchrist, S. (1999), The Financial Accelerator in a Quantitative Business Cycle Framework, *in* J. B. Taylor and M. Woodford, eds, 'Handbook of Macroeconomics', Vol. 1 of *Handbook of Macroeconomics*, Elsevier, chapter 21, pp. 1341–1393.

Boivin, J. and Giannoni, M. (2006), DSGE Models in a Data-Rich Environment, NBER Technical Working Papers 0332, National Bureau of Economic Research, Inc.

Chodorow-Reich, G. (2013), 'The employment effects of credit market disruptions: Firm-level evidence from the 2008–9 financial crisis', *The Quarterly Journal of Economics* **129**(1), 1–59.

Christiano, L., Rostagno, M. and Motto, R. (2010), Financial Factors in Economic Fluctuations, Working Paper Series 1192, European Central Bank.

Del Negro, M., Eusepi, S., Giannoni, M., Sbordone, A., Tambalotti, A., Cocci, M., Hasegawa, R. and Linder, M. H. (2013), The FRBNY DSGE model, Staff Reports 647, Federal Reserve Bank of New York.

Duygan-Bump, B., Levkov, A. and Montoriol-Garriga, J. (2015), 'Financing constraints and unemployment: Evidence from the great recession', *Journal of Monetary Economics* **75**, 89–105.

Gelfer, S. (2019), 'Data-rich dsge model forecasts of the great recession and its recovery', *Review of Economic Dynamics* **32**, 18–41.

Gordon, R. J. (2010), 'The demise of okun's law and of procyclical fluctuations in conventional and unconventional measures of productivity', *Proceedings 100 (May 2010, no. 2)* **11**, 15.

Grant, A. L. (2018), 'The great recession and okun's law', *Economic Modelling* **69**, 291–300.

Justiniano, A., Primiceri, G. and Tambalotti, A. (2011), 'Investment Shocks and the Relative Price of Investment', *Review of Economic Dynamics* **14**(1), 101–121.

Okun, A. M. (1962), Potential gnp: Its measurement and significance in proceedings of the business and economics statistics section, *in* 'American Statistical Association', Vol. 89.

Owyang, M. T. and Sekhposyan, T. (2012), 'Okun's law over the business cycle: was the great recession all that different?', *Federal Reserve Bank of St. Louis Review* **94**(September/October 2012).

Schmitt-Grohé, S. and Uribe, M. (2017), 'Liquidity traps and jobless recoveries', *American Economic Journal: Macroeconomics* **9**(1), 165–204.

Smets, F. and Wouters, R. (2003), 'An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area', *Journal of the European Economic Association* **1**(5), 1123–1175.

Smets, F. and Wouters, R. (2007), 'Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach', *American Economic Review* **97**(3), 586–606.

A Appendix: DSGE-DFM Estimation Algorithm

Following the work of Boivin and Giannoni (2006) and Gelfer (2019), a Metropolis-within-Gibbs algorithm is used to estimate the state space parameters $\Gamma = [\Lambda,\ \Psi,\ R]$ and the structural DSGE parameters θ. The adaptive Metropolis-within-Gibbs algorithm used follows the following steps:

1. Specify Initial values of $\theta^{(0)}$, and $\Gamma^{(0)}$, $\Gamma = \{\Lambda,\ \Psi,\ R\}$

2. Repeat for g=1...G

   2.1 Solve the DSGE model numerically and obtain $G(\theta^{(g-1)})$ and $H(\theta^{(g-1)})$

   2.2 Draw from $p(\Gamma|G(\theta^{(g-1)}), H(\theta^{(g-1)}); X_{1:T})$

      2.2.1 Generate unobserved states $S^{1:T,(g)}$ from $p(S^T|\Gamma^{(g-1)}, G(\theta^{(g-1)}), H(\theta^{(g-1)}); X_{1:T})$ using the Carter-Kohn forward-backward algorithm

      2.2.2 Generate state-space parameters $\Gamma^{(g)}$ from $p(\Gamma|S^{1:T,(g)}; X_{1:T})$ by drawing from a set of known conditional densities $[R|\Lambda, \Psi; S^{1:T,(g)}]$, $[\Lambda|R, \Psi; S^{1:T,(g)}]$, $[\Psi|\Lambda, R; S^{1:T,(g)}]$.

   2.3 Draw DSGE parameters $\theta^{(g)}$ from $p(\theta|\Gamma; X_{1:T})$ using adaptive Metropolis Hastings using the proposal equation of $\theta^* = \theta^{(g-1)} + \bar{c}\,\varepsilon_\ell$ where $\varepsilon_\ell \sim NID(0, \Sigma^{-1})$

      2.3.1 Propose $\theta^* = \theta^{(g-1)} + \bar{c}\,\varepsilon_\ell$ where $\varepsilon_\ell \sim NID(0, \Sigma^{-1})$

      2.3.2 Calculate $P(X_{1:T}|\theta^*, \Gamma^{(g)})$ using the Kalman Filter

      2.3.3 Calculate the acceptance probability ω

$$\omega = min\left\{\frac{P(X_{1:T}|\theta^*, \Gamma^{(g)})P(\theta^*)}{P(X_{1:T}|\theta^{(g-1)}, \Gamma^{(g)})P(\theta^{(g-1)})}, 1\right\}$$

      2.3.4 $\theta^{(g)} = \theta^*$ with probability ω and $\theta^{(g)} = \theta^{(g-1)}$ with probability $(1-\omega)$

   2.4 Calculate acceptance rate of proposed θ for 1 to g draws. If the acceptance rate is lower than target acceptance rate decrease $\bar{c}$ by w (iff $\bar{c} > w$). If acceptance rate is greater than target acceptance rate increase $\bar{c}$ by w. This target acceptance rate adaption can be implemented every n^* iterations of g.

3. Return $\{\theta^{(g)},\ \Gamma^{(g)}\}_{g=1}^{G}$

The priors for the state space parameters include the elements of Λ and the diagonal elements of Ψ and R. The elements of Λ can be separated between core and non-core elements. Core series may only have a single non-zero row element of Λ whose prior is normally distributed and centered around 1. Each non-core series corresponding row elementsof Λ has a multivariate normal prior centered around zero.

The prior for each i^{th} row of the non-core series follows the work of Boivin and Giannoni (2006) and Gelfer (2019), who use a Normal-Inverse-Gamma prior distribution for $(\Lambda_i, R_{i,i})$ so that $R_{i,i} \sim IG_2(.001, 3)$ and the prior mean of factor loadings for the i^{th} row is given by $\Lambda_i|R_{i,i} \sim N(0, R_{i,i}I)$ where the mean is a vector of zeros and I is the identity matrix. The prior for the i^{th} measurement equation's autocorrelation parameter, $\Psi_{i,i}$ is $N(0,1)$ for all rows. The autocorrelation parameter prior is truncated to values inside the unit circle to ensure all error processes are stationary.

B Appendix: Tables and Figures

Figure B.1: Okun's Law for Employment Implied by DSGE-DFM Model

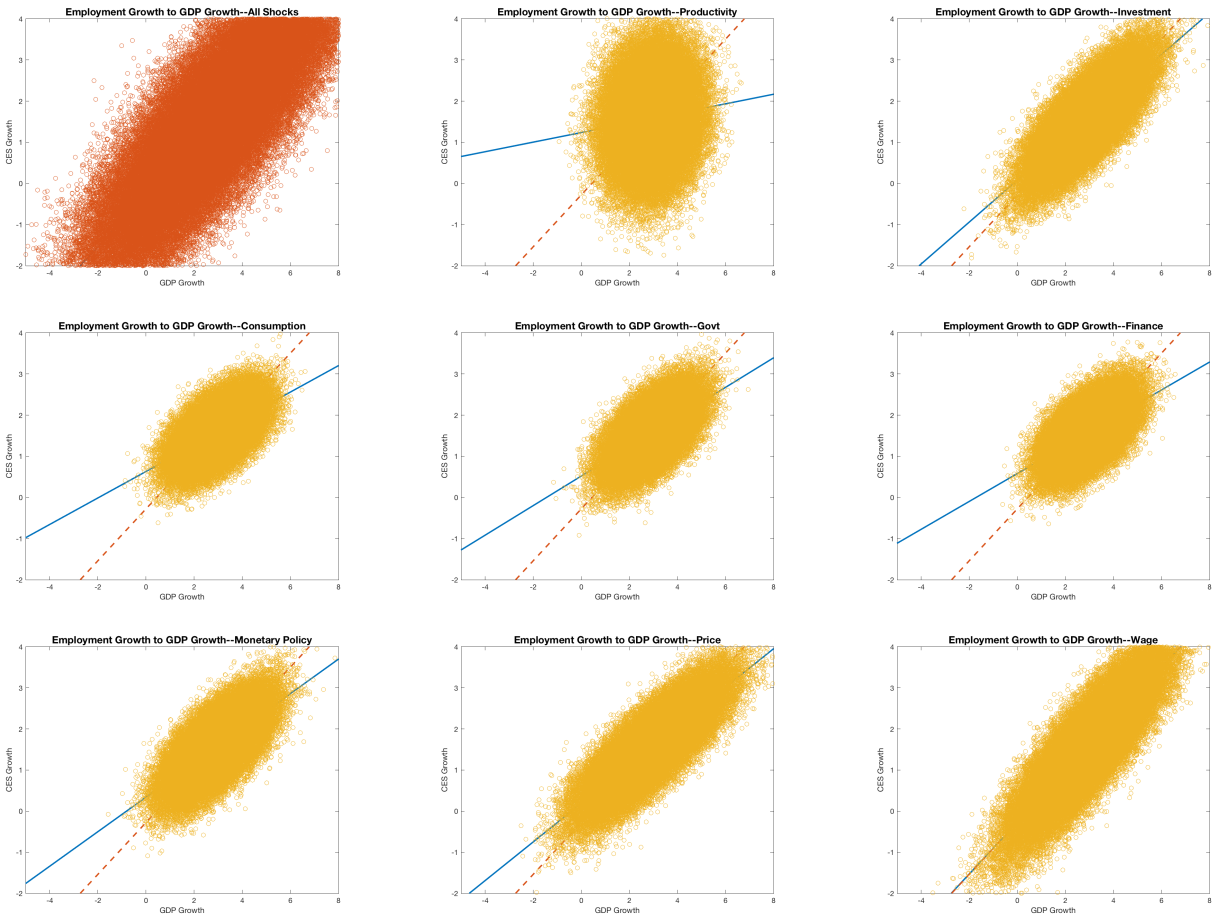


Figure plots the relationship between Employment growth and GDP growth in the DSGE-DFM model, when all shocks are active and when only one specific shock is active in the simulation. The dashed line is a linear regression of Okun's Law for Employment when a specified shock is active and the solid blue line is the fitted linear regression line when all shocks are active in the simulation.