Evaluating the Forecasting Power of an open-economy DSGE model when estimated in a Data-Rich Environment

Sacha Gelfer*

May 2020

Abstract

This paper examines the inferences and forecasting benefits that can be made when one incorporates a large quantity of economic time series into international structural macroeconomic models. I estimate a close variation of Adolfson et al. (2007, 2008) small open-economy dynamic stochastic general equilibrium (DSGE) model in a data-rich environment and evaluate its predictive performance of the Canadian macroeconomy. The data set I use in the paper includes Canadian, American, Asian and European macro-financial data. I compare the forecasting performance of the DSGE model estimated in a data-rich environment (DSGE-DFM) to the forecasts generated by the DSGE model under estimated in its traditional setting and forecasts generated by other reduce form forecasting models. I find that an open-economy DSGE model estimated in a data-rich environment significantly out performs its regularly estimated DSGE counterpart and the DSGE-DFM forecasts that incorporate real-time data are similar or better to the Bank of Canada's Staff Economic Projections for GDP, consumption, investment, and trade statistics. In addition, the DSGE-DFM model of this paper is useful in forecasting both the real and nominal exchange rate in the short and medium-term.

Keywords: DSGE-DFM, Forecasting, Open-economy Macroeconomics, Exchange Rates
JEL: C53, E17, E27, F41, F47

* *Bentley University, Waltham, MA, USA. E-mail address: sgelfer@bentley.edu*

1 Introduction

New open-economy macroeconomics (NOEM) have generated significant empirical and theoretical innovations in international macroeconomic modeling. Estimated dynamic stochastic general equilibrium (DSGE) models became the marquee policy analysis tools for most central banks around the world because of their structural foundations and their competitive forecasting performance against other reduced-form forecasting models such as vector autoregressions (Gurkaynak et al. 2013) and judgement based forecasts by experts (Kolasa et al. 2012). However, many of the DSGE model's forecasts that have been evaluated empirically are centered around a closed-economy and have had significant declines in their predictive power following the Great Recession. (Del Negro and Schorfheide 2013).

Computational gains have led to additional estimation techniques to become possible for medium scale models that were previously thought to be computationally burdensome. One example includes the estimation of DSGE models using a high dimensional data vector first introduced by Boivin and Giannoni (2006) and often referred to as Dynamic Factor Model DSGE estimation or DSGE-DFM estimation for short. Gelfer (2019) found that a closed-economy DSGE model with financial frictions for the United States estimated in the DSGE-DFM fashion was able to significantly out-forecast modern DSGE models not estimated in a data-rich environment and the Survey of Professional Forecasters (SPF) in regard to core macroeconomic growth variables and many labor and financial metrics.

In this paper, I examine the inferences and forecasting benefits that can be made when one incorporates the DSGE-DFM estimation technique of Boivin and Giannoni (2006) and Gelfer (2019) into a small open-economy DSGE model with capital analogous to the model used in Adolfson et al. (2007). The data set I use in the paper to estimate the DSGE model includes 93 series of Canadian, American, Asian and European macro-financial data. To my knowledge, this is the first time an open-economy DSGE model has been estimated in this fashion. Given the construction of traditional open-economy DSGE model estimation (henceforth, DSGE-Reg) researchers were only able to evaluate the forecasts and dynamics of few macroeconomic series, while the DSGE-DFM model of this paper will be able to look at the dynamics and forecasts of such series as domestic employment by sector, domestic credit measures and international financial series.

After estimating the open-economy DSGE-DFM model for Canada, I find that many of the structural parameter estimates are more in line with firm level survey evidence and that many of the exogenous shocks are estimated to be far less volatile and less persistent than the open-economy DSGE-Reg model of this paper. As a result, many of the theoretical foundations and co-movements the structural model is built upon still hold, when we compare the impulse response functions (IRF's) of the DSGE-Reg model to the IRF's of the DSGE-DFM model, but the magnitudes and inertia of the model's variables are very different. This is what will drive the more accurate forecasts and international co-movements that I see in the DSGE-DFM model of this paper.

Open-economy DSGE models have been known to have three drawbacks connected to them. First, the strong theoretical structure entrenched in them has not resulted in improvement in the forecast accuracy for domestic macroeconomic variables. In some cases open-economy DSGE model forecasts of the domestic variables can be less accurate than their closed-economy DSGE model counterpart (Kolasa and Rubaszek, 2018). Second, open-economy DSGE models shed little light on the short and medium term dynamics of real and nominal exchange rates. They often, result in the inability of open-economy DSGE models to out-perform a naive random walk forecasts for the real and nominal exchange rate in the short to medium term for a variety of countries (Ca'Zorzi, Kolasa and Rubaszek, 2017). Third, small open-economy models fail to empirically account for the cross-correlation that exists between the domestic economy and its international trading partners when it comes to the business cycle and price co-movements (Justiniano and Preston, 2010).

The contribution of this paper is to provide an evaluation of whether open-economy DSGE models estimated in a data-rich environment can be successful in mitigating any of these three key issues of open-economy DSGE models that are estimated in the traditional fashion. DSGE-DFM estimation has the potential to address these three as some of the additional data used in the estimation may be informative about the exogenous shocks or other state variables. For example, some exogenous shocks or other state variables, that are assumed to be unobserved, may be partially observed by a combination of other domestic and international data sets. Utilizing the information from additional series could result in more accurate estimations of the model's parameters and states and thus creating more accurate conclusions about the model's dynamics. The key insight of this paper is that in fact, an open-economy DSGE-DFM model can mitigate all three of these issues.

First, like Kolasa and Rubaszek (2018), I find that the open-economy DSGE-Reg model does not produce better forecasts for GDP, consumption, investment, interest rates and inflation when compared to closed-economy reduced form forecasting models or the Bank of Canada's Staff Economic Projections (SEP). However, the open-economy DSGE-DFM model that incorporates real-time domestic and international macro-financial data was able to significantly out-forecast both the SEP and the open-economy DSGE-Reg model for domestic growth variables, exports and imports in regard to both the short and medium term horizon.

Second, in accordance with Ca'Zorzi, Kolasa and Rubaszek (2017), I find that the open-economy DSGE-Reg model of this paper is able to slightly out-forecast a naive random walk for the real exchange rate, but not the nominal exchange rate. However, the open-economy DSGE-DFM model of this paper is able to out-forecast a naive random walk in regard to both the real exchange rate and the nominal exchange rate. In addition, the exchange rate forecasting performance of the DSGE-DFM model of this paper is amplified when it incorporates real-time international macro-financial data that would have been currently available to the econometrician.

Third, in contrast to Justiniano and Preston (2010), I find that there are significant co-movements in the DSGE-DFM model of this paper between the United States and Canada. When I input United States' macro-financial data into the model that corresponds to the empirical average U.S. recession of the last 70 years, I am able to generate the probabilistic co-movements in Canada's macroeconomy. Further, when I evaluate the model implied correlation between the Canadian economy and the U.S. economy, I find that GDP growth and its components do resemble empirical correlations seen in the data. However, the DSGE-DFM model is still not able to produce any policy rate correlation between the United States and Canada and a smaller than empirically observed correlation for inflation between the two countries.

I choose use Canada as the domestic country in the open-economy model for various reasons. First, Canada is included in the forecasting applications of both Kolasa and Rubaszek

(2018) and Ca'Zorzi, Kolasa and Rubaszek (2017). Second, I will be able to directly address the findings of Justiniano and Preston (2010) in this paper. Finally, the extensiveness of the Bank of Canada's SEP collected and analyzed by Champagne, Poulin-Bellisle and Sekkel (2018) allows me to compare the model's forecasts to a real-time expert based forecasts for a number of macroeconomic variables.

The remainder of this paper is structured as follows. Section 2 briefly explains the features of the open-economy DSGE model and presents its linearized equations. Section 3 outlines the estimation technique used to incorporate the large set of international economic and financial series. Section 4 discusses the priors for the state-space and structural parameters as well as an overview of the data series used in the estimation. Section 5 presents the estimation results and the resulting dynamics of the DSGE-DFM model. Section 6 examines the forecasting performance of the DSGE-DFM model against its DSGE-Reg counterpart, other reduced form models, and the Bank of Canada's Staff Economic Projections for various macroeconomic variables. In addition, the forecasts and dynamics of the real and nominal exchange rates in the DSGE-DFM model are compared against random walk forecasts and other small and large data forecasting models. Section 7 considers the impact a U.S. recession that is calibrated using historical averages would have on Canada's macroeconomy. Finally, section 8 concludes and discusses future extensions.

2 The Structural DSGE Model

The structural DSGE model I use in the paper is an extension of the Smets and Wouters (2003, 2007) New Keynesian model as well as the Lubik and Schorfheide (2007) and Adolfson et al. (2007, 2008) open-economy models. It features a domestic sector and a world sector, endogenous capital, price and wage setting frictions and indexation as well as capital utilization and investment adjustment costs. In this section, I outline the agents of the open-economy DSGE model and I present the linearized equations of the model around the steady state that I use to produce my results.

2.1 General Outline of the Model

The model involves a number of exogenous shocks, economic agents, and market frictions. The agents include an exogenous world economy, domestic households, domestic capital producers, domestic firms and government agencies.

World Economy is specified as an exogenous simple New Keynesian micro-founded closed-economy model without capital. The world sector has price stickiness, price indexation, habit persistence and purchases and sells consumption goods from and to the domestic sector. The short term nominal interest rate is determined by the foreign monetary authority, which is assumed to follow a generalized Taylor Rule. The world economy is subject to demand, price and monetary policy shocks. In addition, the world and domestic economy share a permanent technology shock that alters the steady-state growth path of production.

Households supply household-specific labor to employment agencies. Households maximize a CRRA utility function over an infinite horizon with additively separable utility in consumption, leisure and money. Household earn income form wages, rented capital, and interest payments from domestic and foreign bonds. Utility from consumption includes a habit persistence measure. The final consumption good that enters the domestic household utility function is an aggregate of home and foreign goods. Households are subject to an exogenous preference and risk premium shocks that can be viewed as a shock in the consumer's consumption and savings decisions and international bond investment decision.

Firms supply domestic goods and exports in a monopolistically competitive market. Firms produce differentiated goods, decide on labor and capital inputs, and set domestic prices and export prices in a Calvo (1983) manner. As with wages, those firms unable to change their prices, are able to partially index them to past inflation rates. Firms face four types of exogenous shocks, the first is a permanent global productivity shock, the second is a temporary technology shock that affects their production ability. The third is a price mark-up shock which captures the degree of competitiveness in the domestic and foreign goods market as well as the labor input market. Finally firms are also subject to global asymmetric technology shocks.

Capital Producers are competitive firms that control the creation of new capital (Investment), a process that requires both the newly bought consumption output and the previous stock of capital in the economy. The investment procedure is subject to adjustment costs and capital producers are subject to investment shocks that affect the marginal efficiency of investment as in Justiniano et al. (2011).

Domestic Government Agencies are composed of a monetary authority and a fiscal authority. The short term nominal interest rate is determined by the monetary authority, which is assumed to follow a generalized Taylor Rule and is subject to monetary policy shocks. The fiscal authority sets fiscal policy and is subject to exogenous government spending shocks.

2.2 Log Linear Equations

The model is linearized around the non-stochastic steady state and then solved using the Sims (2002) method. This solution is the transition equation in the state-space set-up of Section 3. Variables denoted with a a time script are defined as log deviations around the steady state. Variables denoted without a time script are steady state values. If variables are stationary $\left(y_t = log\left(\frac{Y_t}{Y}\right)\right)$ and if variables are non-stationary $\left(y_t = log\left(\frac{Y_t}{Z_t}\right) - log\left(\frac{Y}{Z}\right)\right)$ where Z_t is the stochastic growth path determined by permanent technology shocks. In all, the open-economy model is reduced to 32 equations and 15 exogenous shocks all of which are listed in this subsection.

2.2.1 World Sector

The world sector IS curve is a combination of the following two equations:

$$\frac{(1-H^*)}{1-\beta H^*}\lambda_t^* = (1-H^*)(z_{y^*,t} - H^*\beta E_t\left[z_{y^*,t-1}\right]) - (1-(\beta H^*)^2)y_t^* \\ + H^*(y_{t-1}^* + \beta E_t\left[y_{t+1}^*\right] - \zeta_t + \beta E_t\left[\zeta_{t+1}\right]) \tag{1}$$

$$\lambda_t^* = R_t^* + E_t\left[\lambda_{t+1}^*\right] - E_t\left[\zeta_{t+1}\right] + E_t\left[\pi_{t+1}^*\right] \tag{2}$$

where λ_t^* is the marginal utility of output in the foreign household utility function, y_t^*, R_t^* and π_t^* is world output, world nominal interest rate and world inflation rate respectively. While ζ_t is a permanent global technology shock and $z_{y^*,t}$ is foreign demand shock. β is the global discount rate and H^* is the habit persistence in foreign output.

The world sector yields a Keynesian Phillps curve and linearized Taylor Rule equal to:

$$\pi_t^* = \frac{1}{1+\chi^*\beta}(\beta E_t\left[\pi_{t+1}^*\right] + \chi^*\pi_{t-1}^* + \kappa^* y_t) + \mu_{\pi^*,t} \tag{3}$$

$$R_t^* = \rho_r^* R_{t-1}^* + (1-\rho_r^*)\left[\phi_{\pi^*}\pi_t^* + \phi_{y^*}y_t^* + \phi_{dy^*}(y_t^* - y_{t-1}^*)\right] + \varepsilon_{R^*,t} \tag{4}$$

where χ^* is world inflation indexation, κ^* is the slope of the world Phillips curve and $\mu_{\pi^*,t}$ and $\varepsilon_{R^*,t}$ are world price and monetary policy shocks.

2.2.2 Resource Identities and Factor Demands

The open-economy's resource constraint takes the form:

$$y_t = C_y c_t + I_y i_t + G_y z_{G,t} + \frac{I_y r^k}{\delta} u_t + nx_t \tag{5}$$

where C_y, I_y and G_y are the steady state ratios of consumption , investment and government expenditures. While r^k is the real rental rate of capital, δ is the depreciation rate and u_t is the utilization rate of capital. nx_t is net exports and $Z_{G,t}$ is an exogenous government purchase shock of domestic goods. The definitions of net exports, exports and imports are all defined below.

$$nx_t = \alpha(ex_t - im_t - t_t) \tag{6}$$

$$ex_t = -\eta P_{H,t} + y_t^* - z_{D,t} \tag{7}$$

$$im_t = -\eta P_{F,t} + C_y c_t + I_y i_t + G_y z_{G,t} + \frac{I_y r_k}{\delta} u_t \tag{8}$$

where t_t is the terms of trade relation, α is the degree of openness to foreign goods and η is the elasticity of substitution between domestically produced and imported goods. $P_{H,t}$ and $P_{F,t}$ denote the relative price for domestic goods and imported goods relative to the final good. $z_{D,t}$ is an export shock the reflects international differences in productivity growth.

The domestic economy's resource constraint and aggregate production takes the following forms:

$$y_t^H = (1-\alpha)\left(-\eta P_{H,t} + C_y c_t + I_y i_t + G_y z_{G,t} + \frac{I_y r_k}{\delta} u_t\right) + \alpha e x_t \tag{9}$$

$$y_t^H = \psi_H k_t + (1-\psi_H)(z_{H,t} + L_t) \tag{10}$$

where $y_t^H$is domestic production and k_t ad L_t are domestic effective capital and labor with ψ_H being the proportion of capital in the domestic production function. Aggregate domestic production is subject to temporary labor augmenting technology shocks denoted by $z_{H,t}$.

The domestic factor input and marginal costs for domestic firms are:

$$r_t^k = w_t + L_t - k_t \tag{11}$$

$$mc_t = (1-\psi_H) w_t + \psi_H r_t^k - (1-\psi_H) z_{H,t} \tag{12}$$

2.2.3 Domestic Investment and Consumption

The relationship between the stock of capital k_t^s and effective capital k_t is

$$k_t = u_t + k_t^s - \zeta_t \tag{13}$$

while the capital accumulation equation is given by

$$k_{t+1}^s = (1-\delta)(k_t^s - \zeta_t) + \delta(i_t + z_{I,t}) \tag{14}$$

where $z_{I,t}$ is an investment shock to the marginal efficiency of capital creation.

The capital utilization condition captures capital utilization costs in the parameter λ_u. Its condition states:

$$r_t^k = \frac{1}{\lambda_u} u_t \tag{15}$$

The linearized investment transition equation is given by:

$$i_t = \frac{1}{1+\beta}\left(\beta E_t\left[i_{t+1} + \zeta_{t+1}\right] + i_{t-1} - \zeta_t + \frac{1}{\lambda_I}(q_t + z_{I,t})\right) \tag{16}$$

where λ_I capture investment adjustment costs. A large λ_I implies that adjusting the investment schedule is costly. q_t is the domestic relative price of capital and is characterized by:

$$q_t = -(R_t - E_t\left[\pi_{t+1}\right]) + (1 - \beta - \beta\delta)E_t\left[r^k_{t+1}\right] + (\beta - \beta\delta)E_t\left[q_{t+1}\right] \tag{17}$$

The linearized consumption transition equation that includes habit persistence (h) and a consumption shock $(z_{C,t})$ is given by:

$$\begin{aligned} c_t = & \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t\left[c_{t+1}\right] - \frac{1-h}{1+h}(R_t - E_t\left[\pi_{t+1}\right]) \\ & -\frac{1-h}{1+h}(E_t\left[z_{C,t+1}\right] - z_{C.t}) + \frac{1}{1+h}E_t\left[\zeta_{t+1}\right] - \frac{h}{1+h}\zeta_t \end{aligned} \tag{18}$$

2.2.4 Trade and Exchange Rate

The real exchange rate (s_t) is given by

$$s_t = s_{t-1} + \pi_{\epsilon,t} + \pi^*_t - \pi_t \tag{19}$$

where $\pi_{\epsilon,t}$ is the nominal depreciation rate of the home currency. The linearized terms of trade relation equation is:

$$t_t = P_{F,t} - P^*_{H,t} - s_t \tag{20}$$

where $P^*_{H,t}$ is the relative price of exports to the world sector in terms of the final good.

The evolution of the domestic country's real net asset position $(b^*_{H,t})$ is:

$$b^*_{H,t+1} = \frac{1}{\beta}b^*_{H,t} + nx_t \tag{21}$$

The linearized uncovered interest rate parity condition is given by:

$$s_t = E_t\left[s_{t+1}\right] - \left(R_t - E_t\left[\pi_{t+1}\right]\right) + \left(R_t^* - E_t\left[\pi_{t+1}^*\right]\right) - \phi_B b_{H,t+1}^* - z_{B_t} \tag{22}$$

where ϕ_B and z_{B_t} are a risk premium scale parameter and exogenous shock respectively.

2.2.5 New Keynesian Philips Curve

The domestic economy has four linearized New Keynesian Phillips curves. One for wage inflation $(\pi_{w,t})$:

$$\pi_{w,t} = \beta E_t\left[\pi_{w,t+1}\right] + \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\psi\epsilon_w)}\Bigg(\psi L_t + \frac{1}{1-h}(c_t - hc_{t-1}) - z_{C,t} + \frac{h}{1-h}\zeta_t - w_t + \mu_{w,t}\Bigg) + \gamma_W(\pi_{t-1} - \beta\pi_t) + (1-\beta\rho_\zeta)\zeta_t \tag{23}$$

where ψ is the CRRA coefficient on Labor in the domestic household's utility function. In addition to wages there is a linearized New Keynesian Phillips curve for domestic price inflation $(\pi_{H,t})$, export price inflation $(\pi_{H,t}^*)$ and import price inflation $(\pi_{F,t})$:

$$\pi_{H,t} = \beta E_t\left[\pi_{H,t+1}\right] + \gamma_H(\pi_{t-1} - \beta\pi_t) + \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}\left(mc_t - P_{H,t} + \mu_{H,t}\right) \tag{24}$$

$$\pi_{H,t}^* = \beta E_t\left[\pi_{H,t+1}^*\right] + \gamma_{H^*}(\pi_{t-1}^* - \beta\pi_t^*) + \frac{(1-\beta\theta_{H^*})(1-\theta_{H^*})}{\theta_{H^*}}\left(mc_t - P_{H,t}^* - s_t + \mu_{H^*,t}\right) \tag{25}$$

$$\pi_{F,t} = \beta E_t\left[\pi_{F,t+1}\right] + \gamma_F(\pi_{t-1} - \beta\pi_t) + \frac{(1-\beta\theta_F)(1-\theta_F)(1-\psi_F)}{\theta_F(1-\psi_F+\psi_F\epsilon_F)}\left(\eta_{mc}y_t^* - P_{F,t} + s_t + \mu_{F,t}\right) \tag{26}$$

where θ_ℓ denotes the degree of price stickiness parameter and γ_ℓ is the degree of price indexation for wages, domestic prices, export prices and import prices. $\mu_{\ell,t}$ is a price mark up shock to a particular market, $\ell = \{w, H, H^*, F\}$.

2.2.6 Price Definitions

The linearized domestic CPI identity is equal to:

$$(1 - \alpha)P_{H.t} + \alpha P_{F,t} = 0 \tag{27}$$

Wage inflation is defined as:

$$\pi_{w,t} - \pi_t = w_t - w_{t-1} + \zeta_t \tag{28}$$

Domestic good inflation is defined as:

$$\pi_{H,t} - \pi_t = P_{H,t} - P_{H,t-1} \tag{29}$$

Export good inflation is defined as:

$$\pi^*_{H,t} - \pi^*_t = P^*_{H,t} - P^*_{H,t-1} \tag{30}$$

Import good inflation is defined as:

$$\pi_{F,t} - \pi_t = P_{F,t} - P_{F,t-1} \tag{31}$$

2.2.7 Monetary Policy and Exogenous Shocks

Monetary policy is conducted using the following linearized Taylor Equation that determines the nominal interest rate:

$$R_t = \rho_r R_{t-1} + (1 - \rho_r)\left[\phi_\pi \pi_t + \phi_y y_t + \phi_s(s_t - s_{t-1})\right] + \varepsilon_{R,t} \tag{32}$$

In all, the the open-economy model has 15 exogenous shocks, 13 of which are AR(1) processes the lone exceptions being the global monetary policy shock and domestic monetary shock which is simply white noise. Included, in the remaining 13 shocks are two foreign sector shocks, four mark-up shocks, three component demand shocks, two technology shocks, a risk premium shock and a global productivity shock that alters the global steady state growth path. All processes are assumed to be i.i.d. with mean zero and standard deviation σ_i and autocorrelation parameters ρ_i, where $i = \{z_y^*, \mu_{\pi^*}, \mu_w, \mu_H, \mu_{H^*}, \mu_F, z_C, z_I, z_G, z_H, z_D, z_B, \zeta, \varepsilon_{R^*}, \varepsilon_R\}$

3 Estimation

I estimate the model in two different ways. The first method estimates the model using specific data that matches particular states in the model, I call this DSGE-Reg estimation. The second method estimates the model using specific data that matches particular states and data that have no direct connection to the model's endogenous variables, I call this method DSGE-DFM estimation. I use the common approach in the literature and estimate each method in two stages. In the first stage I estimate the exogenous world sector's structural parameters. In the second stage, I estimate the small open-economy's structural parameters, taking as given the posterior mean values of the world sector's parameters from the first stage. In the remaining subsections, I give an overview of both the DSGE-Reg and DSGE-DFM estimation methods.

3.1 Regular DSGE Estimation

The state space representation of the solved model consists of a transition equation, which is calculated by solving the linearized system of the given model one wishes to evaluate for a given set of structural model parameters (θ):

$$S_t = G(\theta)S_{t-1} + H(\theta)v_t \quad \text{where } v_t \sim NID(0, I_m) \tag{33}$$

and the measurement equation:

$$X_t^{reg} = \Lambda S_t + \epsilon_t \quad \text{where } \epsilon_t \sim NID(0, R^{reg}) \tag{34}$$

Here X_t^{reg} are the economic data sets and Λ is a matrix matching the observed data to the definitions of the model's state variables S_t.[1]

[1] For more detail on Bayesian DSGE-Reg estimation techniques please see An and Schorfheide (2007).

3.2 DSGE-DFM Estimation

Bayesian estimation of a DSGE model in a data-rich environment incorporates the state space model discussed above with a few modifications.

The set up for DSGE-DFM estimation is characterized by equations (35)-(37).

$$S_t = G(\theta)S_{t-1} + H(\theta)v_t \text{ where } v_t \sim NID(0, I_m) \tag{35}$$

$$X_t = \Lambda S_t + e_t \tag{36}$$

$$e_t = \Psi e_{t-1} + \epsilon_t \text{ where } \epsilon_t \sim NID(0, R) \tag{37}$$

Here e_t follows an AR(1) process and is often referred to as measurement error. The matrix X is J x T where J is the number of data series used in estimation and T is the number of observables for each series. The Matrix Λ is now no longer assumed to be known by the econometrician, but instead is estimated within the MCMC routine. The matrices Ψ and R that govern the measurement error's stationary processes for each series are assumed to be diagonal and are also estimated within the MCMC routine.

The measurement equation (36) has the following structure:

$$\begin{bmatrix} Output \\ Inflation \\ \vdots \\ ------ \\ [Expendture\ Components] \\ [Labor\ Market] \\ [Yields] \\ [Credit\ Market] \\ [Price/Wage\ Indexes] \end{bmatrix} = \begin{bmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ -- & ---- & --- & -- \\ [\lambda_{E_1}] & [\lambda_{E_2}] & ... & [\lambda_{E_n}] \\ [\lambda_{L_1}] & [\lambda_{L_2}] & ... & [\lambda_{L_n}] \\ \vdots & \vdots & ... & \vdots \end{bmatrix} \begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \\ \vdots \\ \epsilon_t^f \end{bmatrix} + \begin{bmatrix} e_{t,1} \\ e_{t,2} \\ \vdots \\ e_{t,J} \end{bmatrix}$$

where X_t is partitioned into core series and non-core series separated by the dashed line. The core series are series that are only allowed to load on one particular variable of the state vector, S_t, to which there is a known sole relationship between series and state. (For instance, GDP to Y) Further, the factor loading coefficient for the first series of each core variable that corresponds to a particular known state is assumed to be perfectly tight, this is represented by the 1's in the Λ matrix. This anchors the estimated states of the DSGE model and ensures that they don't drift too far away from their theoretical foundation.

The non-core series consists of the remaining data sets not in the core series and are grouped into subgroups. These series are allowed to "load" on all time t states in the state vector. Non-core series may have up to n (where n is the number of elements in S_t) non-zero elements for their corresponding row in Λ unlike the core series whose corresponding row in Λ may only have one non-zero element.

Following the work of Boivin and Giannoni (2006) and Gelfer (2019), a Metropolis-within-Gibbs algorithm is used to estimate the state space parameters $\Gamma = [\Lambda, \Psi, R]$ and the structural DSGE parameters θ. The adaptive Metropolis-within-Gibbs algorithm used follows the following steps:

1. Specify Initial values of $\theta^{(0)}$, and $\Gamma^{(0)}$, $\Gamma = \{\Lambda, \Psi, R\}$

2. Repeat for g=1...G

   2.1 Solve the DSGE model numerically and obtain $G(\theta^{(g-1)})$ and $H(\theta^{(g-1)})$

   2.2 Draw from $p(\Gamma|G(\theta^{(g-1)}), H(\theta^{(g-1)}); X_{1:T})$

      2.2.1 Generate unobserved states $S^{1:T,(g)}$ from $p(S^T|\Gamma^{(g-1)}, G(\theta^{(g-1)}), H(\theta^{(g-1)}); X_{1:T})$ using the Carter-Kohn forward-backward algorithm

      2.2.2 Generate state-space parameters $\Gamma^{(g)}$ from $p(\Gamma|S^{1:T,(g)}; X_{1:T})$ by drawing from a set of known conditional densities $[R|\Lambda, \Psi; S^{1:T,(g)}]$, $[\Lambda|R, \Psi; S^{1:T,(g)}]$, $[\Psi|\Lambda, R; S^{1:T,(g)}]$.

   2.3 Draw DSGE parameters $\theta^{(g)}$ from $p(\theta|\Gamma; X_{1:T})$ using adaptive Metropolis Hastings using the proposal equation of $\theta^* = \theta^{(g-1)} + \bar{c}\,\varepsilon_\ell$ where $\varepsilon_\ell \sim NID(0, \Sigma^{-1})$

   2.4 Calculate acceptance rate of proposed θ for 1 to g draws. If the acceptance rate is lower than target acceptance rate decrease $\bar{c}$ by w (iff $\bar{c} > w$). If acceptance rate is greater than target acceptance rate increase $\bar{c}$ by w. This target acceptance rate adaption can be implemented every n^* iterations of g. In addition the condition $w \to 0$ as $g \to \infty$ must be satisfied

3. Return $\{\theta^{(g)},\ \Gamma^{(g)}\}_{g=1}^{G}$

The applied algorithm is based on 200,000 draws (2 parallel chains of 100,000 draws discarding the initial burn-in period of 25,000 iterations). Σ^{-1} is the inverse Hessian of the DSGE model evaluated at its posterior mode under regular estimation. The calibrations regarding the adaptive step include the acceptance target rate which is set at 27%, an initial $\bar{c}$ which is set to .1, the adaptive jump size w which is set at .005 and an adjustment rate n which is set at 25. The adjustment rate n^* determines how many iterations take place between changing $\bar{c}$ as described in step 2.4.[2]

4 Data and Priors

In total I estimate the model using 93 Canadian and international quarterly data series that cover the time period of 1995Q1 t0 2017Q4.[3]. The first stage (World sector) is estimated with 3 data series in the DSGE-Reg estimation and 31 in the DSGE-DFM estimation method. For estimating the entire open-economy DSGE-Reg model a total of 16 data series are used and a total of 93 data series are used to estimated the entire open-economy DSGE-DFM model.

Let's first discuss the series used in estimating the DSGE-Reg and DSGE-DFM structural parameters of the exogenous world sector. Table 1 lists the different data sets used in estimating the world sector.

To calculate world aggregates, I use a trade volume weighted average of Canada's five largest trading partners and the EU from 2000 to 2017. The weights and countries used to determine the world aggregates are listed in Table 2. The world aggregate policy rate does not include the Bank of China's policy rate.

[2]For more detail on the Bayesian DSGE-DFM estimation technique please see Gelfer (2019).
[3]A 3-month average is used to obtain quarterly data from monthly series.

Table 1: Data Series used in Estimation of the World Sector

Data Set	Transform	Data Set	Transform
DSGE-Reg Estimation			
World Aggregate GDP Growth	0		
World Aggregate Inflation	0		
World Aggregate Policy Rate	β		
DSGE-DFM Estimation			
World Aggregate GDP Growth	0	United States 10-year Treasury	0
World Aggregate Inflation	0	Germany 10-year Government Bond Rate	0
World Aggregate Policy Rate	β	UK 10-year Government Bond Rate	0
United States GDP Growth	0	France 10-year Government Bond Rate	0
EU GDP Growth	0	Italy 10-year Government Bond Rate	0
China GDP Growth	0	Spain 10-year Government Bond Rate	0
Mexico GDP Growth	0	Switzerland 10-year Government Bond Rate	0
Japan GDP Growth	0	Japan 10-year Government Bond Rate	0
United States Inflation	0	United States Major Stock Index Growth	0
EU Inflation	0	China Major Stock Index Growth	0
China Inflation	0	Japan Major Stock Index Growth	0
Mexico Inflation	0	Mexico Major Stock Index Growth	0
Japan Inflation	0	EU ETF Stock Index Growth	0
United States Policy Rate	0		
EU Policy Rate	0		
Japan Policy Rate	0		
UK Policy Rate	0		
Mexico Policy Rate	0		

Table 2: Countries and Weights used to determine the World Sector Aggregates

Country	Weight on World Sector
United States	.753
EU	.122
China	.061
Mexico	.027
Japan	.025
South Korea	.012

Table 3 list all the data used for the Canadian DSGE-Reg and DSGE-DFM models. In all the DSGE-DFM model is estimated using the 16 core data series in the DSGE-Reg model while also adding sub-aggregate GDP expenditure components, total and sub-aggregate employment data, various yields and yield spreads, credit market statistics and multiple measure of inflation to its X_t data set. As is common in the Dynamic Factor Model literature, all non-core series sample standard deviation is normalized to 1. Measurement equations for all core series and the transformation code applied to each series can be found in Appendix A.

Table 3: Data Series used in Estimation of the Canadian DSGE-DFM model

Data Set	Transform	Data Set	Transform
DSGE-DFM Estimation			
Core-Series (Used in DSGE-Reg Estimation)			
World Aggregate GDP Growth	0	CAN Import Growth	γ
World Aggregate Inflation	0	CAN Hours Growth	0
World Aggregate Policy Rate	β	CAN Wage Inflation	0
CAN GDP Growth	γ	CAN Export Inflation	0
CAN Consumption Growth	γ	CAN Import Inflation	0
CAN Investment Growth	γ	CAN CPI Inflation	0
CAN Government Growth	γ	CAN Real FX Rate Growth	0
CAN Export Growth	γ	CAN Policy Rate	β
International Series			
United States GDP Growth	0	Mexico Policy Rate	0
EU GDP Growth	0	United States 10-year Treasury	0
China GDP Growth	0	Germany 10-year Government Bond Rate	0
Mexico GDP Growth	0	UK 10-year Government Bond Rate	0
Japan GDP Growth	0	France 10-year Government Bond Rate	0
United States Inflation	0	Italy 10-year Government Bond Rate	0
EU Inflation	0	Spain 10-year Government Bond Rate	0
China Inflation	0	Switzerland 10-year Government Bond Rate	0
Mexico Inflation	0	Japan 10-year Government Bond Rate	0
Japan Inflation	0	United States Major Stock Index Growth	0
United States Policy Rate	0	China Major Stock Index Growth	0
EU Policy Rate	0	Japan Major Stock Index Growth	0
Japan Policy Rate	0	Mexico Major Stock Index Growth	0
UK Policy Rate	0	EU ETF Stock Index Growth	0
Expenditure Components			
CAN Industrial Production	1	CAN Residential Investment	1
CAN Durable Cons Goods	1	CAN Non-Residential Structure Inv	1
CAN Semi-Durable Cons Goods	1	CAN Machinery and Equipment Inv	1
CAN Non-Durable Cons Goods	1	CAN Intellectual Property Inv	1
CAN Consumption Services	1	CAN Inventory Investment	1
CAN Business Investment	1		

Data Set	Transform	Data Set	Transform
Employment			
CAN Total Employees	1	CAN Professional Services Employees	1
CAN Agricultural Employees	1	CAN Business Services Employees	1
CAN FFMO Employees	1	CAN Educational Services Employees	1
CAN Utilities Employees	1	CAN Health care Services Employees	1
CAN Construction Employees	1	CAN Informational Services Employees	1
CAN Manufacturing Employees	1	CAN Food Services Employees	1
CAN Wholesale and Retail Employees	1	CAN Public Employees	1
CAN Transportation Employees	1	CAN Unemployment Rate	0
CAN FIRE Employees	1		
Yields and Spreads			
Consumer Loan Rate	0	CAN 2-year to 3-month Treasury Spread	0
Prime Corporate Rate	0	CAN 5-year to 2-year Treasury Spread	0
Overnight Money Market Rate	0	CAN 7-year to 2-year Treasury Spread	0
CAN 3-month Treasury Rate	0	CAN 10-year to 2-year Treasury Spread	0
CAN 6-month to 3-month Spread	0	CAN Mortgage Rate	0
CAN 1-year to 3-month Treasury Spread	0		
Credit Markets			
CAN Total Business Credit Outstanding	1	CAN Outstanding Business Bonds	1
CAN Total Business Loans Outstanding	1	CAN Mortgage Credit Outstanding	1
CAN Total Commercial Paper	1	CAN Consumer Credit Outstanding	1
CAN Housing Starts	2		
Other Prices			
CAN Core CPI Inflation	0	USD/CAD Exchange Rate Growth	0
CAN GDP Deflator Inflation	0	Oil Price Inflation	0

4.1 Structural and State-Space Parameter Priors

The structural parameter marginal priors are in accordance to ToTemII by Dorich et al. (2013) priors. Some structural parameters are fixed including the global discount rate, share of capital, depreciation rate, and the steady state share of consumption, government, investment and imports to total Canadian output. The latter parameters being calibrated to the average proportion of consumption, government investment and import purchases of Canadian GDP over the sample period. β is calibrated to correspond to a steady state nominal interest rate of 3%, the average of the Canadian policy rate over the sample period. γ is calibrated to correspond to an annual per capita GDP growth rate of 1.5%, roughly the average of World per capita GDP over the sample period. A complete list of calibrated structural parameters as well as the prior mean, standard deviation and description of the estimated structural parameters can be found in Table B.1 and Table B.2.

The priors for the state space parameters include the elements of Λ and the diagonal elements of Ψ and R. The elements of Λ can be separated between core and non-core elements. Core series may only have a single non-zero row element of Λ whose prior is normally distributed and centered around 1. Each non-core series corresponding row elements[4] of Λ has a multivariate normal prior centered around zero.

The prior for each i^{th} row of the non-core series follows the work of Boivin and Giannoni (2006) and Gelfer (2019), who use a Normal-Inverse-Gamma prior distribution for $(\Lambda_i, R_{i,i})$ so that $R_{i,i} \sim IG_2(.001, 3)$ and the prior mean of factor loadings for the i^{th} row is given by $\Lambda_i | R_{i,i} \sim N(0, R_{i,i} I)$ where the mean is a vector of zeros and I is the identity matrix. The prior for the i^{th} measurement equation's autocorrelation parameter, $\Psi_{i,i}$ is $N(0, 1)$ for all rows. The autocorrelation parameter prior is truncated to values inside the unit circle to ensure all error processes are stationary. All prior distributions for the state space parameters are summarized in Table 4.

Table 4: Priors for DSGE-DFM Γ Parameters

	Description	Distribution	Mean	Std
Γ Parameters				
$\Psi_{i,i}$	AR(1) coef. of measurement error	Normal	0	1
$R_{i,i}$	Variance of measurement error	Inv. Gamma	0.001	3*
$\Lambda_{i,j}$	Factor loadings of Non-core data sets	Normal	0	$R_{i,i}I$
$\Lambda_{i,j}$	Factor loadings of Core data sets	Normal	1	0

5 Estimation Results

This section is divided between estimation results and the economic inferences that DSGE-DFM estimation can tell us about how economic series and sectors not directly modeled in the structural DSGE model react to structural economic shocks. Posterior estimates of the structural parameter are discussed first. Second, impulse response functions (IRF's) of the world sector and domestic sector are presented and discussed along with the diagnostic inferences they bring when comparing economic events that are driven by different types of

[4]The elements of Λ that correspond to $t-1$ states of the S_t vector are assumed to be zero.

structural shocks. This will help us understand why the DSGE-DFM model is able to create superior forecasts compared to other models for so many macroeconomic variables.

The posterior estimates for the structural parameters for both estimation techniques are tabulated in Tables B.3 and B.4. A few observations emerge. First, in the world sector the persistence of world productivity growth shocks is estimated to be almost non-existent in the DSGE-DFM model compared to the DSGE-Reg model. With mean ρ_ζ estimated to be 0.21 in the DSGE-DFM model compared to 0.85 in the DSGE-Reg model. This implies that changes in the steady state growth path of output are much less persistent but more volatile when a model is estimated in a data-rich environment. Additionally, the New Keynesian Phillips curve is estimated to be flatter in the DSGE-DFM model. All remaining parameters in the world sector show little difference between the two estimation techniques.

The price and wage Calvo estimates for wages, domestic prices and import prices are lower in the DSGE-DFM model, suggesting less nominal rigidities in pricing behavior. These smaller, yet still significant, price and wage rigidities are more in line with the findings of Amirault et al. (2006) who surveyed monthly price changes by business type and found that the median price duration is about every 3 months. However, the price stickiness of exports is estimated to be much larger in the DSGE-DFM model, suggesting exports are more likely to face pricing frictions.

The parameters that govern habit formation substantially increase in the DSGE-DFM estimation for the world sector decrease in the domestic sector when compared to their estimates under DSGE-Reg estimation. Additionally, elasticity of substitution between home and domestic goods is estimated at a higher level in the DSGE-DFM model. Taylor Rule policy parameters are found to be more responsive to the exchange rate when estimated in the data-rich environment. All of these findings will account for different trade, inflation and consumption dynamics when we compare the core macroeconomic variables between the two estimation techniques.

Many of the parameters linked to the exogenous shocks of the model become smaller across the estimation techniques of the DSGE model. The standard deviation coefficients on consumption and investment shocks are estimated to be significantly lower in the DSGE-DFM technique. Price and wage mark-up shocks are estimated to be slightly less persistent, but have much lower volatility estimates in the DSGE-DFM estimation technique. The presence of other price and wage indexes, including oil prices, may drive this result as different inflation dynamics are needed to describe them.

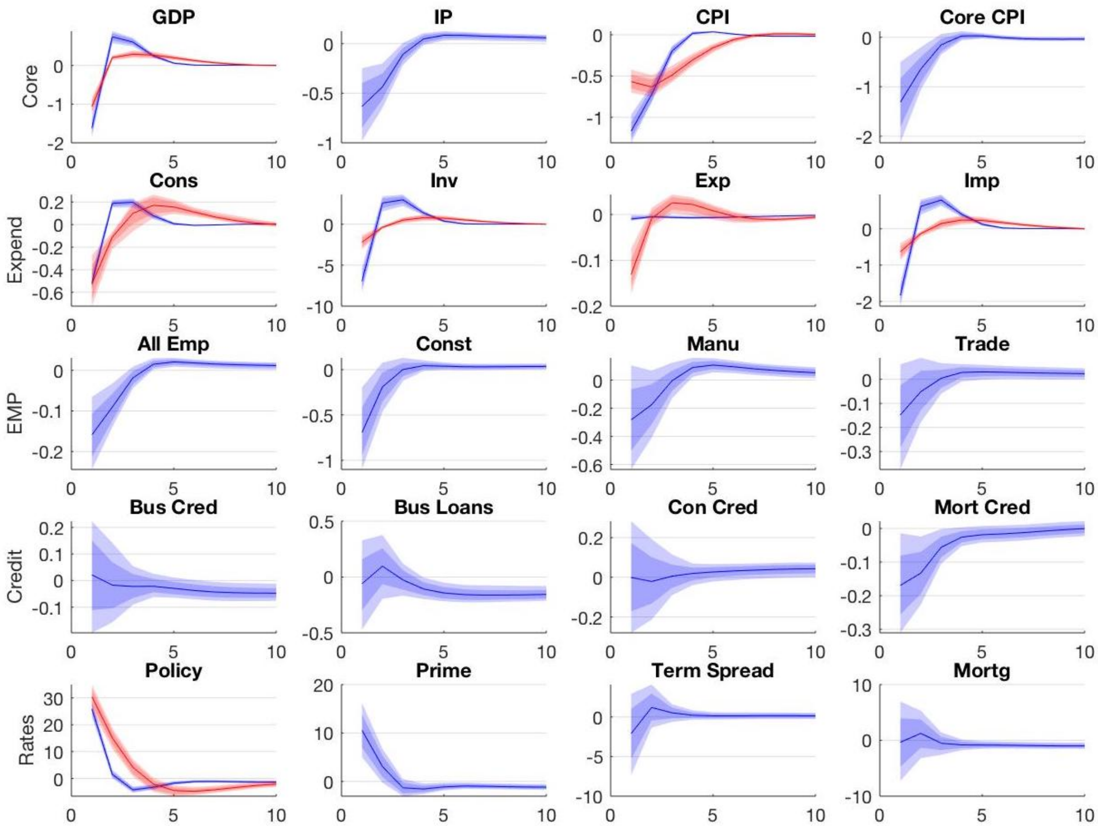
5.1 Impulse Response Functions

In this subsection, I illustrate some of the key economic mechanisms at work in the model's equilibrium. I do so with the aid of impulse response functions (IRF's) shocking the economy, which I report in Figures 1 to 7. DSGE-DFM estimation allows for economic series not directly corresponding to any endogenous variables in the DSGE model to be related to the model's exogenous shocks and endogenous variables. This allows IRF's to be generated for many economic series whose IRF's do not exist outside of structural VAR estimation. The figures plot the median IRF and 90% posterior density band for the DSGE-DFM model in blue and the median IRF and 90% posterior density band for the DSGE-Reg model in red. All expenditure growth rates and inflation are in annualized growth rates. Employment and credit measures plot the percentage deviation away from their linear trends and all rates and spreads plot the change in basis points.

Figure 1 presents responses to a monetary policy shock that increases the nominal domestic interest rate. There is little difference in core variables between the estimation techniques with the interest rate rising by 25 basis point and GDP and CPI inflation both falling as a result. Employment in the construction sector contracts the most as mortgage rates rise and outstanding mortgage credit falls. Due to the different structural parameters between the estimation techniques Investment and GDP fall by a grater extent in the DSGE-DFM model.

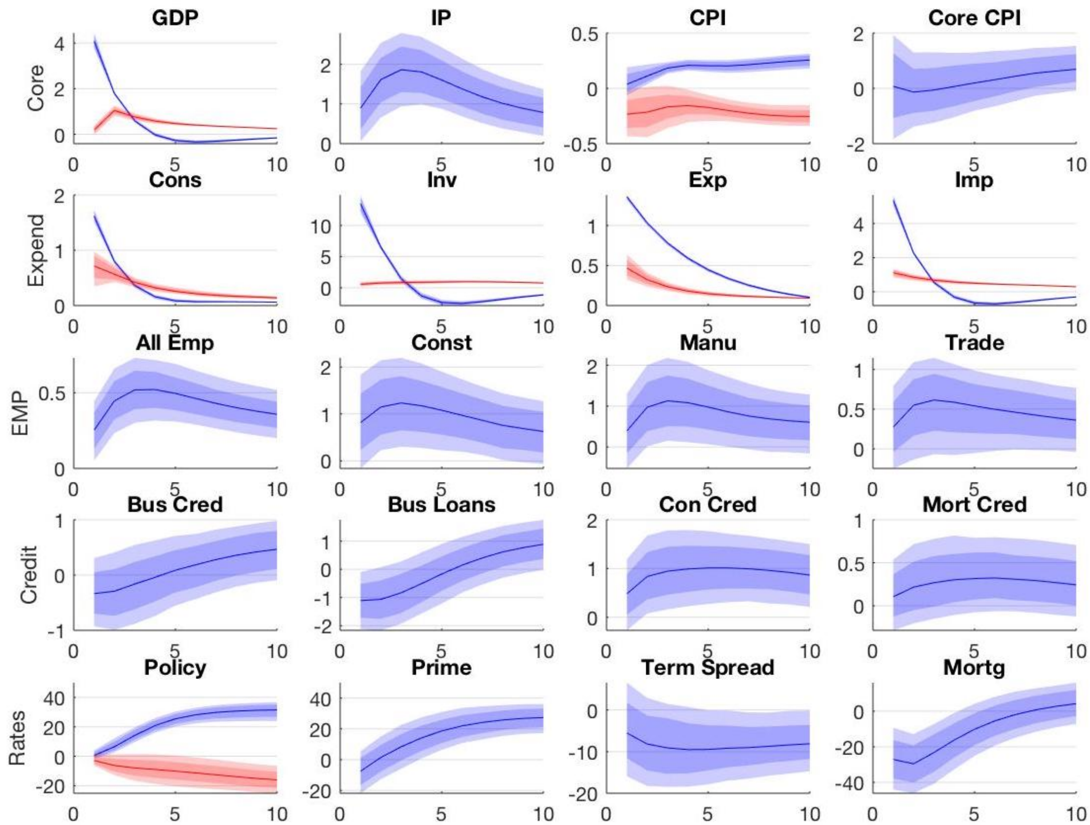
Figure 2 plots responses to a global productivity shock that temporarily increase the steady state growth path. The higher estimated σ_ζ and lower estimated ρ_ζ in the DSGE-DFM model creates bigger increases and faster returns to steady state growth in GDP and expenditure growth from the shock compared to the DSGE-Reg model. As theory would suggest, employment increases across all sectors and household credit expands while business credit falls slightly. One difference between the two estimated models is that the policy rate decreases from the shock in the DSGE-reg model while it increases in the DSGE-DFM model.

Figure 1: Domestic Monetary Policy Shock (ε_R)



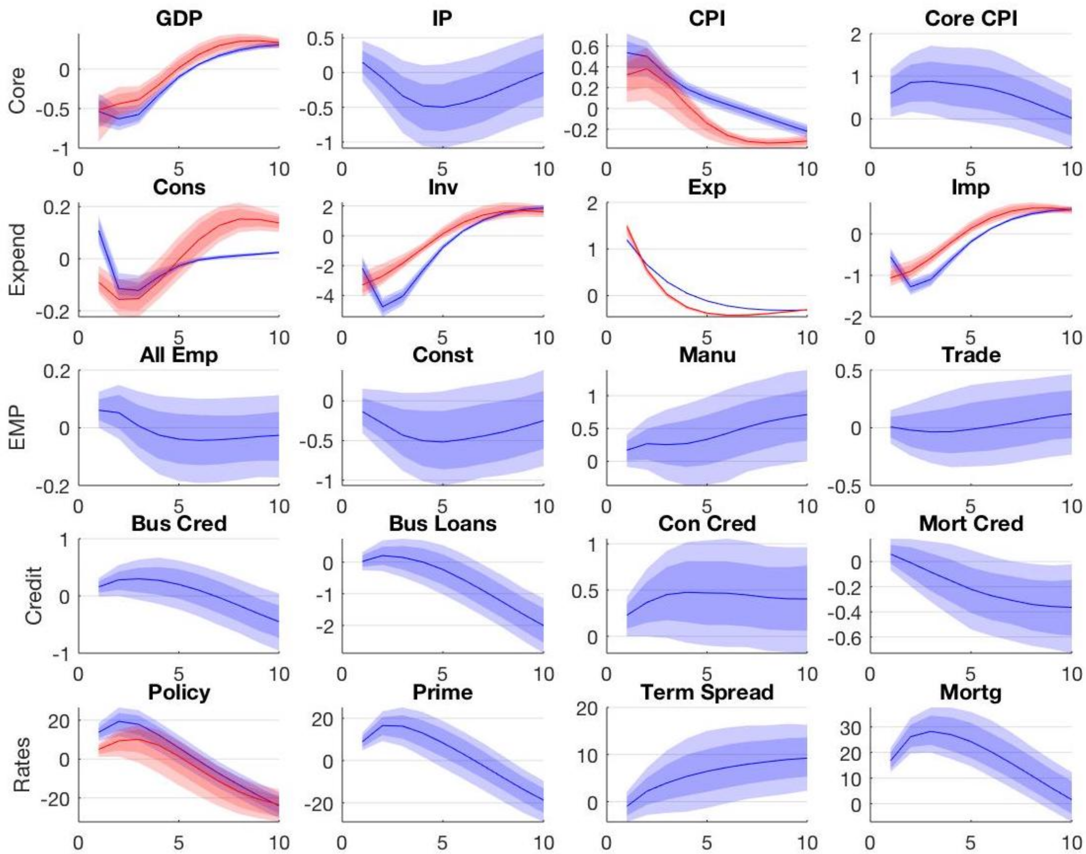
As shown in Figure 3, a positive demand shock in the worlrd sector increases domestic exports and inflation in both models. The higher inflation causes the monetary authority to raise interest rates, decreasing domestic consumption and investment. The resulting export boom, higher policy rate and higher mortgage rate causes employment to rise in the manufacturing sector and employment to fall in the construction sector resulting in a neutral movement in total employment.

Figure 2: Global Productivity Shock (ζ)



A positive consumption shock plotted in Figure 4 provides the classic trade off of consumption and investment in both models. Domestic inflation is less impacted by the positive consumption shock in the DSGE-DFM model partly due to the lower θ_H so the policy rate stays unchanged in the DGSE-DFM model compared to an increase in the DSGE-Reg model. The consumption shock corresponds to an increase in consumer credit, imports and employment in the retail trade sector as we would expect.

Figure 3: World Demand Shock (z_{y^*})



There is little difference between the models with respect to an export shock as illustrated in Figure 5. Domestic inflation is more stable again in part to a lower θ_H in the DSGE-DFM model causing the policy rate to fall by a lesser amount relative to the policy rate fall in the DSGE-Reg model. Industrial Production increases by a greater amount than does GDP after the positive export shock. Similar results are found when we look at IRF's generated by an investment shock plotted in Figure 6.

Figure 4: Consumption Shock (z_C)

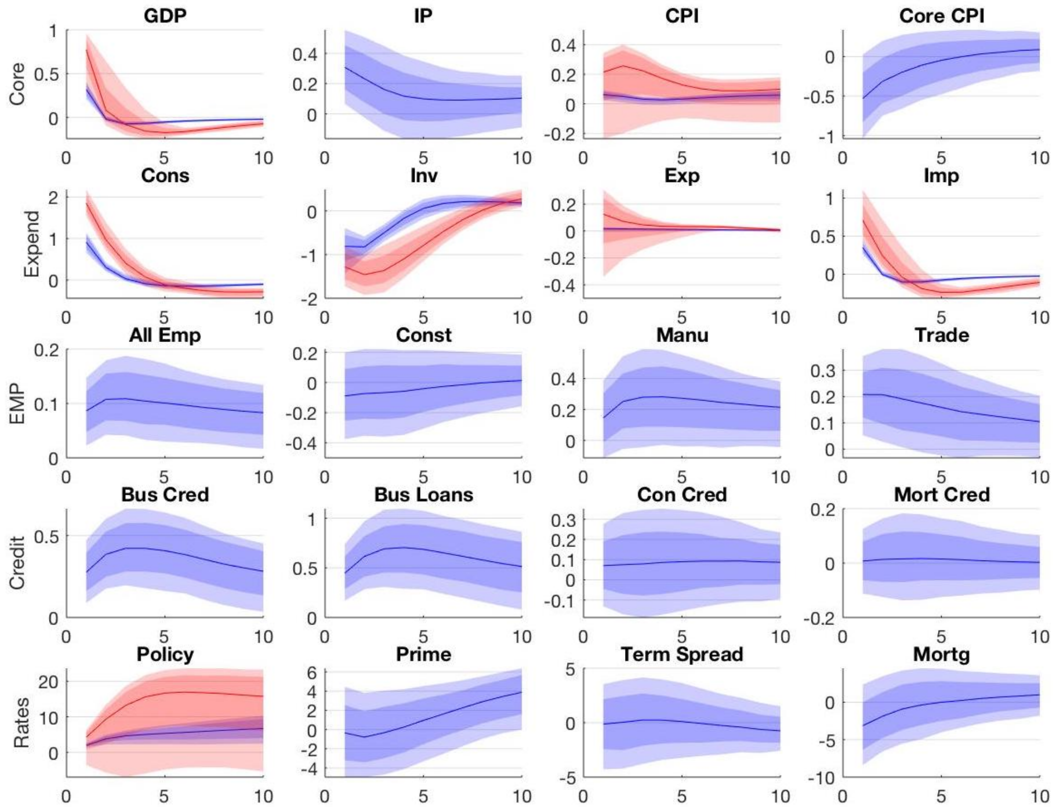


Figure 5: Export Shock (z_D)

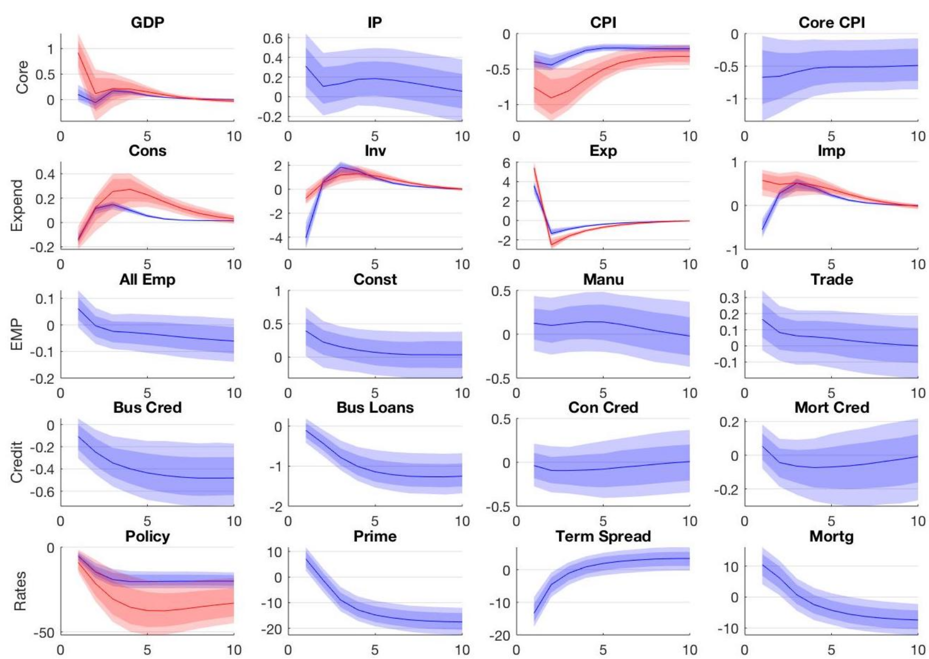


Figure 6: Investment Shock (z_I)

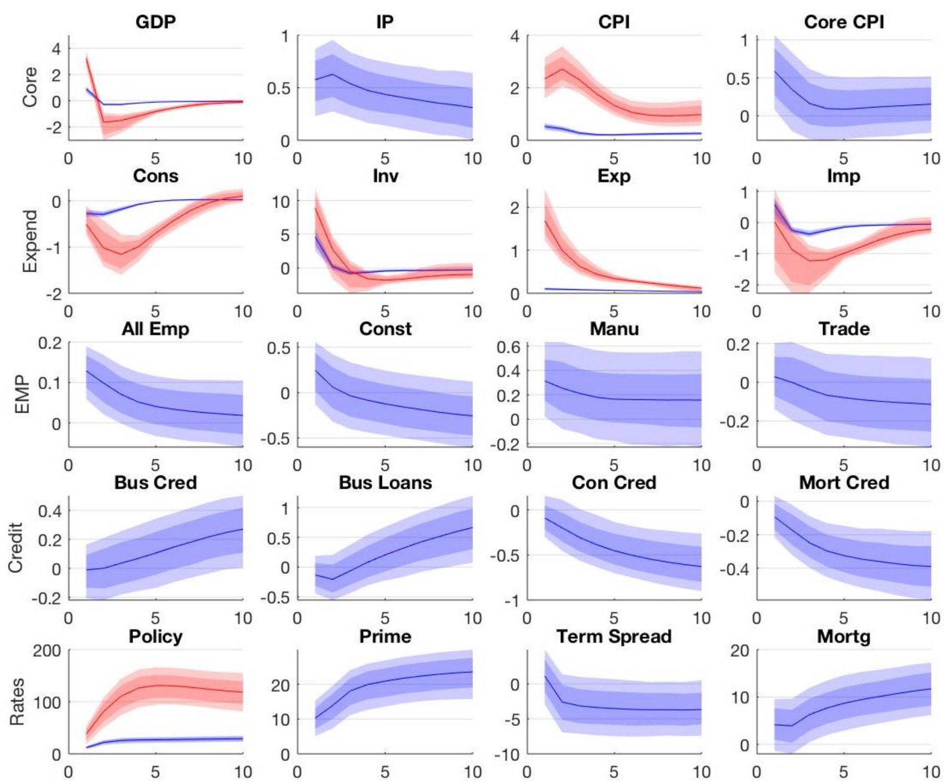
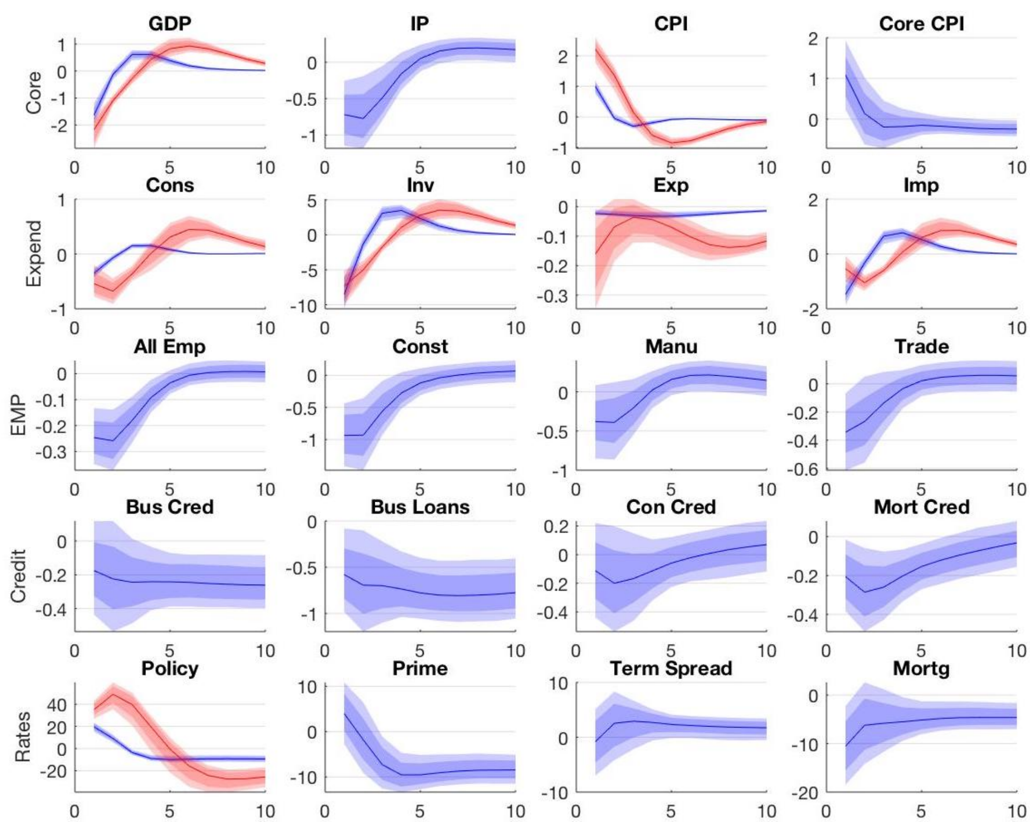


Figure 7 plots responses to a positive domestic price shock. The positive domestic price shock causes GDP and Industrial production to fall and inflation and interest rates to rise. Employment falls in the capital intensive sectors of construction and manufacturing as investment falls by a larger amount than does consumption in both models. Conversely, employment in wholesale and retail trade falls by a smaller amount when compared to employment in the construction and manufacturing sectors.

Figure 7: Domestic Price Shock (μ_H)



The remaining IRF's of the other eight exogenous shocks can be found in Appendix B along with the IRF's of the world sector that also plot the responses for international countries' macroeconomy from exogenous world shocks.

6 Forecast Evaluation

In this section, I perform a similar exercise as Ca'Zorzi et al. (2017), Kolasa and Rubaszek (2018) and Champagne, Poulin-Bellisle and Sekkel (2018) of comparing the forecast errors of the different models and of the Bank of Canada's Staff Economic Projections (SEP). I compare the accuracy of the forecasts of the SEP to the DSGE model by using root mean squared errors (RMSE) calculated using equations 38 and 39 for x equal to GDP and expenditure component growth as well CPI, Core CPI and the Policy Rate.

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_{t+\tau}^{Forecast} - x_{t+\tau}^{Actual})^2} \tag{38}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left[\left(\frac{\sum_{h=1}^{4} x_{t+h}^{Forecast}}{4}\right) - \left(\frac{\sum_{h=1}^{4} x_{t+h}^{Actual}}{4}\right)\right]^2} \tag{39}$$

Each forecast is generated by 500,000 simulations, 5,000 draws from the posterior parameter distribution and each parameter draw is simulated using 100 draws of future structural shocks for 8 quarters. In all simulations, the zero lower bound is established using shadow monetary policy shocks using an algorithm outlined by Holden and Paetz (2012).

The forecast evaluation window looks at the accuracy of forecasts from 2007Q4 to 2013Q4. I first estimate the models from 1995-2007Q3 and generate forecasts for 2007Q4. When the new actualized data are revealed in 2007Q4, the new values are inserted into the Kalman filter and are used to forecast 2008Q1. The process continues until the model is re-estimated once a year when all Q3 data for a given year would have been available to the econometrician.

I first estimate 4 different model specifications and compare their forecasting accuracy to the forecasts from the SEP. The first is the DSGE-Reg model we have discussed previously. The second is a DSGE-Reg model that calibrates the world sector's structural parameters to be equal to their estimates when the world sector is estimated in a data rich environment. I denote this model as the DSGE-Reg-hybrid. The third model is a DSGE-DFM model estimated only using international data and the domestic core data series, denoted DSGE-

World-DFM and the last is the DSGE-DFM model previously discussed. The relative RMSE for different variables and different time horizons of the pseudo out-of-sample forecast are presented and discussed below.

Table 5 reports the relative RMSE by using equations 38 and 39 and normalizing them against the RMSE for the SEP for different τ, the four quarter ahead rolling forecast average, and 8 quarter ahead rolling forecast average for real GDP, consumption, investment, and government expenditure growth, while Table 6 reports the relative RMSE for real export and import growth, CPI inflation and the Canadian Policy rate. Any relative RMSE above one implies that the model's RMSE is above the RMSE for the SEP and any number below one implies that the model's RMSE is lower than the RMSE for the SEP. The actual RMSE's of the SEP for all variables are reported in Table B.5.

A few patterns emerge. First, the DSGE-Reg and DSGE-Reg-hybrid models have significantly[5] higher RMSE compared to the data-rich DSGE models and the SEP for all variables besides consumption growth for τ inside a year. For $\tau = 6$ and beyond the RMSE for the DSGE-reg models are on par with the SEP forecasts. This is a finding consistent with Del Negro and Schorfeide (2013), Gervais and Gosselin (2014) and Gelfer (2019) where DSGE-Reg models for the United States and Canada can compete with many reduced-form forecast models for $\tau > 4$ but cannot for $\tau \leq 4$.

Introducing the international data sets into the estimation method (DSGE-World-DFM) outlined in Table 3 shrinks the RMSE for all growth variables compared to the DGSE-Reg methods but is still significantly higher than the RMSE of the SEP forecasts for all horizons of τ when it comes to export/import growth, policy and inflation forecasts. However, the RMSE's for investment and consumption growth are significantly decreased and even lower than the SEP's RMSE with the extra data in estimation for $\tau \leq 4$ and the four quarter average of $\tau = 1$.

When the Canadian data series are introduced into the estimation method (DSGE-DFM), the RMSE's are lower for export growth and government growth and remain similar to the RMSE's of the DSGE-World-DFM model for all other variables for $\tau \leq 4$. The introduction

[5]Diebold-Mariano test statistics are computed for various model comparisons and can be found in Tables B.6 and B.7 of the appendix.

of the Canadian data series brings the RMSE's for the average four quarter growth rate of the second year of forecasts below the RMSE's of the SEP for all growth components. Even given the declines of the RMSE's for growth variables, the RMSE does not fall for the forecasted Bank of Canada policy rate when the international series are introduced into the estimation method and the RMSE's remain significantly higher than the SEP's for CPI inflation.

Further, the higher RMSE for the DSGE-Reg-hybrid model suggest that simply calibrating structural parameters that have been determined in a data-rich environment does not significantly increase the forecasting capability of a DSGE model. Instead, the data-rich data set must be included in the estimation routine in order for the DSGE model to reap the forecasting advantages found in Gelfer (2019) and this paper.

6.1 Real-time Forecasting Performance

In addition to the DSGE-DFM models outperforming the DSGE-Reg models and many of the SEP's forecasts throughout the forecast evaluation window (2007Q4-2013Q4), I also see that the "predictive performance enhancing" effects of large data sets used in DSGE-DFM estimation can be amplified if available real time data are used when forecasting with the DSGE-DFM model.

I introduce real-time forecasting using two different approaches. The first approach, denoted DSGE-DFM-RT, assumes that the econometrician would use all real-time financial data (international and domestic) available for the current quarter. For example, in generating the 2007Q4 forecasts the econometrician would know all non-financial data realizations up to and including 2007Q3 and all financial data up to and including 2007Q4.

The second approach, denoted DSGE-DFM-RT 2, uses the same assumption but also assumes that the econometrician would have the first monthly value for all data series in the model that are reported at a monthly frequency. The econometrician would then forecast the remaining two monthly observations for all monthly frequency variables using a simple AR(2) forecasting model to generate the quarterly observation for all monthly frequency variables. For example, in generating the 2007Q4 forecasts the econometrician would know all non-financial data realizations up to and including 2007Q3 and all financial data up to

Table 5: Relative RMSE for CAN Expenditure Growth

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP								
DSGE-Reg	3.40	2.71	1.35	**0.99**	1.14	2.75	1.05	2.12
DSGE-Reg-hybrid	3.64	2.54	1.23	**0.99**	1.35	2.52	1.13	1.82
DSGE-World-DFM	1.65	1.50	**0.98**	**0.92**	1.00	1.20	**0.93**	**0.98**
DSGE-DFM	1.68	1.54	**0.99**	**0.90**	1.03	1.28	**0.86**	1.07
Consumption								
DSGE-Reg	**0.49**	**0.51**	1.20	1.39	1.25	**0.79**	1.45	1.53
DSGE-Reg-hybrid	**0.45**	**0.42**	**0.99**	1.16	1.09	**0.56**	1.18	1.07
DSGE-World-DFM	**0.48**	**0.46**	**0.90**	**0.81**	**0.56**	**0.55**	**0.59**	**0.64**
DSGE-DFM	**0.51**	**0.47**	**0.84**	**0.80**	**0.63**	**0.55**	**0.58**	**0.51**
Investment								
DSGE-Reg	1.11	1.15	1.13	**0.97**	**0.97**	1.17	**0.91**	1.17
DSGE-Reg-hybrid	1.12	1.12	1.17	1.06	1.11	1.17	1.22	1.42
DSGE-World-DFM	**0.91**	**0.87**	**0.97**	**0.99**	**0.98**	**0.77**	1.02	**0.83**
DSGE-DFM	**0.97**	**0.92**	**0.99**	**0.92**	**0.94**	**0.92**	**0.80**	**0.84**
Government								
DSGE-Reg	**0.73**	1.08	1.00	**0.61**	**0.80**	**0.98**	**0.61**	**0.77**
DSGE-Reg-hybrid	**0.71**	1.05	1.00	**0.64**	**0.83**	**0.96**	**0.65**	**0.81**
DSGE-World-DFM	**0.74**	1.12	1.05	**0.67**	**0.87**	1.04	**0.70**	**0.90**
DSGE-DFM	**0.63**	**0.99**	**0.96**	**0.63**	**0.85**	**0.85**	**0.65**	**0.75**

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are more accurate than the SEP.

and including 2007Q4 and all monthly frequency variables up to 2007Q3 and the monthly observation for October 2007. They would then generate forecasts for November and December 2007 for a forecasted observation for 2007Q4. The non-financial monthly frequency variables included in the DSGE-DFM model include all Canadian employment series, credit series, Industrial production, CPI and core CPI.

The relative RMSE's for all models using real-time data are reported in Tables 7 and 8. I include the relative RMSE's for the DGE-DFM model without any real-time data for comparison. First, the RMSE's are lowered for both real-time models for all variables for $\tau \leq 4$ and remain the same for all $\tau > 4$. This suggest that the addition of the real-time data affect only the very short-run dynamics of the structural DSGE model, while leaving

Table 6: Relative RMSE for CAN Trade Growth, CPI, and Policy Rate

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
Exports								
DSGE-Reg	1.57	1.41	1.06	1.05	**0.91**	1.37	1.15	1.40
DSGE-Reg-hybrid	1.53	1.41	1.07	1.05	**0.90**	1.37	1.14	1.44
DSGE-World-DFM	1.34	1.38	1.07	1.06	**0.91**	1.35	1.12	1.44
DSGE-DFM	1.27	1.31	1.06	1.05	**0.92**	1.24	1.15	1.37
Imports								
DSGE-Reg	1.34	1.17	1.15	**0.98**	1.09	1.13	1.00	1.16
DSGE-Reg-hybrid	1.32	1.11	1.13	1.02	1.16	1.04	1.14	1.24
DSGE-World-DFM	1.27	1.12	1.10	1.02	1.00	1.06	1.05	1.09
DSGE-DFM	1.29	1.16	1.09	**0.96**	**0.99**	1.13	**0.90**	**0.97**
Policy Rate								
DSGE-Reg	5.58	2.24	1.46	1.51	1.57	1.79	1.53	1.58
DSGE-Reg-hybrid	6.67	2.82	1.89	1.81	1.82	2.33	1.82	1.94
DSGE-World-DFM	6.79	2.36	1.12	**0.89**	**0.80**	1.66	**0.86**	1.03
DSGE-DFM	7.34	2.61	1.45	1.28	1.17	1.96	1.26	1.41
CPI								
DSGE-Reg	2.78	1.44	1.56	1.95	2.25	2.73	4.07	5.54
DSGE-Reg-hybrid	2.99	1.63	1.83	2.20	2.56	3.27	4.66	6.59
DSGE-World-DFM	2.23	**0.99**	**0.73**	**0.71**	**0.78**	1.24	1.06	1.31
DSGE-DFM	2.34	1.11	1.11	1.19	1.32	1.74	2.36	3.16

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are more accurate than the SEP.

the dynamics for periods 4 through 8 relatively unchanged.

In addition, the relative RMSE's of the DSGE-DFM-RT 2 model are lowered for almost every variable and time horizon measure. The additional data used in the estimation and the use of real-time data have the biggest impact in reducing the RMSE's for investment growth, export growth and import growth. With the addition of the real-time data the RMSE's of the DSGE-DFM-RT 2 are lower than the RMSE of the SEP for consumption, investment, government, export and import growth for all τ. Further, the addition of the real-time data significantly decrease the RMSE's for the policy rate, CPI inflation and core CPI inflation. Finally, the RMSE for one period ahead forecasted GDP growth are lowered by almost 80% compared to the RMSE of the DSGE-reg model but are still higher then the RMSE of the

SEP.

Table 7: Relative RMSE for CAN Expenditure Growth with Real-time Data

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP								
DSGE-DFM	1.68	1.54	**0.99**	**0.90**	1.03	1.28	**0.86**	1.07
DSGE-DFM-RT	1.73	1.32	**0.99**	**0.90**	1.00	1.13	**0.86**	1.01
DSGE-DFM-RT 2	1.22	1.25	**0.97**	**0.90**	**0.99**	1.09	**0.86**	1.01
Consumption								
DSGE-DFM	**0.51**	**0.47**	**0.84**	**0.80**	**0.63**	**0.55**	**0.58**	**0.51**
DSGE-DFM-RT	**0.50**	**0.44**	**0.87**	**0.80**	**0.63**	**0.50**	**0.58**	**0.50**
DSGE-DFM-RT 2	**0.45**	**0.43**	**0.86**	**0.79**	**0.64**	**0.49**	**0.58**	**0.49**
Investment								
DSGE-DFM	**0.97**	**0.92**	**0.99**	**0.92**	**0.94**	**0.92**	**0.80**	**0.84**
DSGE-DFM-RT	**0.71**	**0.89**	**0.99**	**0.93**	**0.91**	**0.78**	**0.81**	**0.81**
DSGE-DFM-RT 2	**0.52**	**0.86**	**0.96**	**0.94**	**0.91**	**0.78**	**0.83**	**0.80**
Government								
DSGE-DFM	**0.63**	**0.99**	**0.96**	**0.63**	**0.85**	**0.85**	**0.65**	**0.75**
DSGE-DFM-RT	**0.71**	1.00	**0.97**	**0.63**	**0.85**	**0.91**	**0.65**	**0.79**
DSGE-DFM-RT 2	**0.65**	**0.99**	**0.98**	**0.63**	**0.86**	**0.88**	**0.65**	**0.78**

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are more accurate than the SEP.

Table 8: Relative RMSE for CAN Trade Growth, CPI, and Policy Rate with Real-time Data

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
Exports								
DSGE-DFM	1.27	1.31	1.06	1.05	**0.92**	1.24	1.15	1.37
DSGE-DFM-RT	**0.91**	**0.99**	**0.85**	**0.82**	**0.93**	**0.95**	**0.87**	1.08
DSGE-DFM-RT 2	**0.86**	**0.98**	**0.84**	**0.82**	**0.92**	**0.95**	**0.88**	1.08
Imports								
DSGE-DFM	1.29	1.16	1.09	**0.96**	**0.99**	1.13	**0.90**	**0.97**
DSGE-DFM-RT	**0.84**	**0.89**	**0.87**	**0.68**	**0.98**	**0.90**	**0.83**	**0.90**
DSGE-DFM-RT 2	**0.80**	**0.90**	**0.87**	**0.68**	**0.98**	**0.91**	**0.82**	**0.91**
Policy Rate								
DSGE-DFM	7.34	2.61	1.45	1.28	1.17	1.96	1.26	1.41
DSGE-DFM RT	1.12	1.56	1.13	1.08	1.02	1.26	1.06	1.09
DSGE-DFM-RT 2	1.26	1.58	1.14	1.08	1.02	1.26	1.06	1.09
CPI								
DSGE-DFM	2.34	1.11	1.11	1.19	1.32	1.74	2.36	3.16
DSGE-DFM-RT	1.44	1.14	1.02	1.07	1.30	1.57	2.16	2.73
DSGE-DFM-RT 2	1.22	1.12	1.03	1.07	1.28	1.52	2.16	2.71
Core CPI								
DSGE-DFM	1.73	1.16	1.07	1.28	1.56	1.29	1.58	1.71
DSGE-DFM-RT	1.46	1.24	1.03	1.14	1.43	1.26	1.41	1.53
DSGE-DFM-RT 2	1.35	1.21	1.04	1.15	1.43	1.19	1.42	1.51

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are more accurate than the SEP.

6.2 Comparing DSGE-DFM to other Forecasting Models

In addition to comparing the forecasts generated by the DSGE-DFM model against the SEP forecasts, I also evaluate the forecast accuracy of the DSGE-DFM forecasts to other reduced form forecasting models. During the great moderation, structural closed-economy DSGE models of the United States were found to out-forecast Bayesian VARs in terms of short-term and medium term output, inflation and short term interest rates (Edge and Gurkaynak, 2010, Gurkaynak et al. 2013 and Wolters 2015).

However, during the Great Recession and its recovery this result seems to have only held for certain DSGE models, most notably the closed economy DSGE models with financial frictions (Del Negro et al. 2013 and Cai et al. 2019). Further, Gelfer (2019) found that closed-economy DSGE-DFM models that have financial frictions amplify the forecasting accuracy of DSGE models around the financial crisis and the Great Moderation for all key macroeconomic variables outside of inflation. In this subsection, I compare the forecasting results of the open-economy DSGE-DFM and DSGE-Reg models against forecasts generated by reduced form time-series models to see if the above mentioned results hold for an international DSGE-DFM model.

Like the previous literature I choose to compare the DSGE-DFM model to standard VARs of the form of equation 40.

$$X_t^{VAR} = \sum_{i=1}^{n} \hat{\Lambda}_i X_{t-i}^{VAR} + \hat{e}_t \tag{40}$$

where n is the number of lags and X_t^{VAR} encompasses the same sixteen data series used to estimate the DSGE-Reg model of this paper.

I also choose to add a dynamic factor model (DFM) to the model evaluation pool because Stock and Watson (2011) have found that over the 3-6 month time horizon DFMs out perform many simple and more complex forecasting models. The principle behind a DFM is that there exists a handful of latent factors f_t inside the economy that power the co-movements among macroeconomic variables. These latent factors are believed to be extractable using a large set of macroeconomic time series.

I use the DFM linear/Guassian state space set-up of equations (41-42) outlined in Stock and Watson (2011) to estimate the parameters of the DFM model.

$$X_t^{DFM} = \hat{\lambda} f_t + \hat{e}_t \tag{41}$$

$$f_t = \Psi f_{t-1} + \omega_t \tag{42}$$

where N is the number of series used in estimation and q is the number of extracted latent factors and $\hat{\lambda}$ is a Nxq matrix of factor loadings. The qxq transition matrix, Ψ, oversees the VAR dynamics of the q latent factors. The 93 series of X_t^{DFM} are identical to the series used in the DSGE-DFM model of this paper. As is common in the Dynamic Factor Model literature, all series sample standard deviations are normalized to 1.

The relative RMSE's for forecasts of the open-economy DSGE-Reg model, vector autoregession model, dynamic factor model, dynamic factor model using real-time data and the open-economy DSGE-DFM model using real time data are reported in Tables 9 and 10. I see that similar to previous literature the DSGE-Reg model of this paper produces similar or better forecasts, in terms of accuracy when compared to forecasts that are produced by a VAR for GDP, its expenditures, inflation and the policy rate. This result is magnified as τ gets larger. However, the RMSE's are still significantly higher than the SEP forecasts for all $\tau \leq 6$ for these variables.

Evaluating the accuracy of the forecasts generated by the dynamic factor model, I see that they are more accurate than both the VAR and DSGE-Reg forecasts for all domestic growth variables, domestic inflation and the domestic policy rate. This is true for all measures of τ. These results are amplified when forecasts from the dynamic factor model incorporate real-time financial and monthly data for the current quarter. However, the forecasts generated by the open-economy DSGE-Reg model for export and import growth are determined to be more accurate than those generated by the DFM model with or without real-time data. This suggests that the theoretical structure of the open-economy DSGE-Reg model is providing insight into the future dynamics of international trade variables. Nevertheless, the RMSE's produced by the forecasts of the DFM and DFM RT 2 models are still higher than the RMSE of the SEP for GDP, exports, imports and the policy rate for most τ's.

Finally, I see that the RMSE's of the open economy DSGE-DFM model that utilizes real-time data are the lowest compared to all of the models and for all variables besides inflation in regard to all values of τ. In addition, the relative RMSE for the DSGE-DFM RT 2 model are below one for consumption, investment, government purchases, exports, imports for all τ and GDP for $\tau > 2$. This signifies that the open-economy DSGE-DFM model produces more accurate forecasts than those produced by the expert based SEP. However, the DSGE-DFM model is not able to forecast the dynamics associated with domestic inflation, domestic core inflation or the policy rate as well as the SEP does. In fact the only model, that is able to out-forecast the SEP for $\tau > 1$ when it comes to domestic inflation and core inflation is the DFM model that utilizes real time data.

Table 9: Relative RMSE for CAN Expenditure Growth with Real-time Data

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP								
DSGE-Reg	3.40	2.71	1.35	**0.99**	1.14	2.75	1.05	2.12
VAR(2)	2.55	2.89	1.12	1.06	1.02	2.11	1.36	2.02
DFM	1.69	1.87	1.10	**0.99**	1.03	1.65	1.06	1.44
DFM-RT 2	1.64	1.71	1.05	**0.95**	**0.99**	1.47	**0.98**	1.23
DSGE-DFM-RT 2	1.22	1.25	**0.97**	**0.90**	**0.99**	1.09	**0.86**	1.01
Consumption								
DSGE-Reg	**0.49**	**0.51**	1.20	1.39	1.25	**0.79**	1.45	1.53
VAR(2)	**0.99**	**0.89**	1.26	1.19	**0.72**	1.12	**0.84**	1.42
DFM	**0.65**	**0.62**	1.02	**0.89**	**0.60**	**0.78**	**0.69**	**0.97**
DFM-RT 2	**0.61**	**0.59**	**0.97**	**0.87**	**0.64**	**0.70**	**0.71**	**0.89**
DSGE-DFM-RT 2	**0.45**	**0.43**	**0.86**	**0.79**	**0.64**	**0.49**	**0.58**	**0.49**
Investment								
DSGE-Reg	1.11	1.15	1.13	**0.97**	**0.97**	1.17	**0.91**	1.17
VAR(2)	1.81	1.89	1.42	1.24	1.29	1.34	1.38	1.63
DFM	**0.91**	1.07	1.07	**0.97**	**0.96**	1.11	**0.92**	1.01
DFM-RT 2	**0.79**	**0.89**	1.07	**0.99**	**0.97**	**0.89**	1.04	**0.93**
DSGE-DFM-RT 2	**0.52**	**0.86**	**0.96**	**0.94**	**0.91**	**0.78**	**0.83**	**0.80**
Government								
DSGE-Reg	**0.73**	1.08	1.00	**0.61**	**0.80**	**0.98**	**0.61**	**0.77**
VAR(2)	**0.85**	1.39	**0.87**	**0.69**	**0.92**	**0.89**	**0.57**	**0.66**
DFM	**0.67**	**0.98**	**0.91**	**0.62**	**0.82**	**0.83**	**0.62**	**0.74**
DFM-RT 2	**0.70**	**0.99**	**0.93**	**0.61**	**0.81**	**0.86**	**0.61**	**0.75**
DSGE-DFM-RT 2	**0.65**	**0.99**	**0.98**	**0.63**	**0.86**	**0.88**	**0.65**	**0.78**

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are more accurate than the SEP.

Table 10: Relative RMSE for CAN Trade Growth, CPI, and Policy Rate with Real-time Data

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
Exports								
DSGE-Reg	1.57	1.41	1.06	1.05	**0.91**	1.37	1.15	1.40
VAR(2)	2.47	1.72	1.42	1.11	**0.83**	1.89	1.52	2.51
DFM	1.32	1.54	1.18	1.14	**0.99**	1.62	1.47	2.08
DFM-RT 2	1.16	1.29	1.11	1.09	**0.97**	1.29	1.31	1.63
DSGE-DFM-RT 2	**0.86**	**0.98**	**0.84**	**0.82**	**0.92**	**0.95**	**0.88**	1.08
Imports								
DSGE-Reg	1.34	1.17	1.15	**0.98**	1.09	1.13	1.00	1.16
VAR(2)	1.17	1.67	1.61	1.14	1.06	1.57	1.29	2.01
DFM	1.42	1.45	1.23	1.00	**0.92**	1.43	**0.93**	1.39
DFM-RT 2	1.13	1.21	1.22	1.03	**0.94**	1.19	1.02	1.20
DSGE-DFM-RT 2	**0.80**	**0.90**	**0.87**	**0.68**	**0.98**	**0.91**	**0.82**	**0.91**
Policy Rate								
DSGE-Reg	5.58	2.24	1.46	1.51	1.57	1.79	1.53	1.58
VAR(2)	8.80	3.30	1.61	1.26	1.16	2.30	1.23	1.45
DFM	9.80	2.36	1.21	1.05	**0.91**	1.67	**0.99**	1.09
DFM-RT 2	5.87	2.08	1.28	1.09	**0.95**	1.65	1.05	1.19
DSGE-DFM-RT 2	1.26	1.58	1.14	1.08	1.02	1.26	1.06	1.09
CPI								
DSGE-Reg	2.78	1.44	1.56	1.95	2.25	2.73	4.07	5.54
VAR(2)	3.08	1.18	**0.82**	1.02	**0.94**	1.42	1.56	2.05
DFM	2.30	1.02	**0.92**	**0.84**	**0.88**	1.46	1.44	2.12
DFM-RT 2	2.16	**0.97**	**0.93**	**0.81**	**0.91**	1.31	1.40	1.94
DSGE-DFM-RT 2	1.22	1.12	1.03	1.07	1.28	1.52	2.16	2.71
Core CPI								
DFM	1.62	**0.96**	**0.86**	**0.86**	**0.87**	**0.87**	**0.80**	**0.78**
DFM-RT 2	1.63	**0.94**	**0.85**	**0.84**	**0.86**	**0.84**	**0.76**	**0.71**
DSGE-DFM-RT 2	1.35	1.21	1.04	1.15	1.43	1.19	1.42	1.51

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are more accurate than the SEP.

6.3 Forecasts of the Real and Nominal Exchange Rate

It has been shown by Ca'Zorzi, Kolasa and Rubaszek (2017) that open-economy DSGE models provide little predictive power when it comes to the short and medium term dynamics of real and nominal exchange rates. These models often do not posses the ability to out-forecast a naive random walk for the real and nominal exchange rate in the short to medium term. In this subsection, I compare the forecasts of reduced form models, open-economy DSGE-Reg and DSGE-DFM models against the naive random walk (RW) forecast for both the real and nominal exchange rate.

The relative RMSE's for forecasts of the open-economy DSGE-Reg model, vector autoregession model, dynamic factor model, dynamic factor model using real-time data and the open-economy DSGE-DFM model not utilizing and utilizing real time data are reported in Table 11. Any relative RMSE above one implies that the model's RMSE is above the RMSE for the RW benchmark and any number below one implies that the model's RMSE is lower than the RMSE for the RW benchmark.

Examining the table we are able to see the Ca'Zorzi, Kolasa and Rubaszek (2017) finding that an open-economy DSGE-Reg model for Canada is able to out-forecast the RW benchmark for the real exchange rate in regards to $\tau > 2$.[6] The DSGE-Reg model also posses more accurate dynamics of the real exchange rate when compared to the reduced-form models of the VAR and dynamic factor model. In comparison the open-economy DSGE-DFM model that does not utilize real-time data is able to beat the benchmark RW for all τ in regards to the real exchange rate. A result that is only amplified when real-time data is utilized. The dynamic factor model also outperforms the RW benchmark for the real exchange rate for $\tau < 4$ when it utilizes real-time data but fails to hold its forecasting gains for $\tau \geq 4$ like the open-economy DSGE-DFM model does.

In regard to the nominal exchange rate[7] a similar pattern holds. The open-economy DSGE-Reg model is at best comparable to the RW benchmark for $\tau < 4$. However, the open-economy DSGE-DFM model is able to out-forecast the RW benchmark when it comes

[6] Similar result was found for open economy DSGE-Reg model for the EU area by Christoffel et al (2010).

[7] The nominal exchange rate is defined as the USD/CAD for models that incorporate that dataset. For model that do not (DSGE-Reg and VAR) the nominal exchange rate is defined as $(1 + \%\Delta RER_t + \pi_t^* - \pi_t)(USD/CAD)_{t-1}$

to the nominla exchange rate for all τ. When the open-economy DSGE-DFM model utilizes real-time data the results are significantly[8] amplified and additionally out-forecast the SEP forecasts for the nominal exchange rate for $\tau > 2$. A finding that gives open-economy DSGE-DFM models credibility when it comes to their accuracy in foretelling the future path of real and nominal exchange rates.

Table 11: Relative RMSE for the real and nominal exchange rates

	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 6$	$\tau = 8$
Real Exchange Rate						
VAR(2)	1.15	1.41	1.49	1.37	1.53	1.69
DFM	1.10	1.17	1.22	1.26	1.39	1.55
DFM-RT 2	**0.17**	**0.69**	**0.91**	1.02	1.18	1.27
DSGE-Reg	1.05	1.00	**0.94**	**0.88**	**0.82**	**0.84**
DSGE-DFM	**0.96**	**0.96**	**0.93**	**0.88**	**0.80**	**0.86**
DSGE-DFM-RT 2	**0.18**	**0.60**	**0.78**	**0.82**	**0.81**	**0.84**
Nominal Exchange Rate						
VAR(2)	1.15	1.45	1.55	1.46	1.67	1.83
DFM	1.10	1.18	1.24	1.29	1.47	1.62
DFM-RT 2	**0.10**	**0.68**	**0.90**	1.02	1.20	1.29
DSGE-Reg	1.06	1.01	1.00	**0.95**	**0.87**	**0.85**
DSGE-DFM	**0.94**	**0.93**	**0.90**	**0.84**	**0.77**	**0.90**
DSGE-DFM-RT 2	**0.05**	**0.56**	**0.72**	**0.76**	**0.78**	**0.88**
SEP	**0.05**	**0.54**	**0.79**	**0.95**	1.10	1.05

Notes: The table shows the ratios of the RMSE from a given model in comparison to the forecasts generated by a random walk model. Values below one indicate that forecasts from the model are more accurate than the RW benchmark.

7 Economic Impact of a U.S. Recession

A major advantage to the DSGE-DFM estimation technique is that it permits us to consider the economic effects of structural shocks outside the scope of the standard variables of GDP, prices, and short-term interest rates. Further, given the amount of data series and the international scope of the data set used, DSGE-DFM estimation can help us estimate the impact international, country-specific recessions and financial volatility have on the Canadian

[8] Diebold-Mariano test statistics are computed for various model comparisons against the random walk and can be found in Tables B.8 of the appendix.

economy.

The model is able to quantify the average effect that certain international events would have on the Canadian economy if we assume a starting point for the DSGE-DFM model and input the average macroeconomic estimates of certain global events. For example, we can assume the model is at steady state, calculate the average effects of U.S. data during the start of a U.S. recession to estimate the most likely combination of global shocks to push the U.S. economy to the calibrated recession state. The estimated combination of global shocks can then be used to calculate the IRF's of the Canadian economy from the combination of global shocks. In many ways it is a reverse IRF. Instead of shocking the model away from steady state and seeing how the data responds, I exogenously shock the global data and use the kalman filter to calculate how the global structural shocks "responded".

To create an "Average U.S. Recession", I calculate the median effect of GDP growth, change in inflation, change in the U.S. policy rate, change in the long run interest rate and the return of the U.S. stock index during the first 6 months of the ten post-WWII recessions as determined by the NBER. Table 12 reports the estimates.

GDP contracts by 2.2%, inflation measured by the GDP deflator falls by 45 basis points, the policy rate is cut by 130 basis points, the 10 year U.S. Treasury falls by 51 basis points, and the stock market index contracts by 1.5%. These are the values I input into the fictional X^{World} data set to calculate the filtered structural shocks[9] of the world sector to push the world sector out of a steady state and into a state where GDP growth is 4.69% below steady state, inflation, the policy rate, and the 10-year treasury are 45, 130, and 51 basis points away from steady state respectively and the U.S. stock index growth is 4% below steady state.

Table 12: Median Statistics of Post-WWII U.S. Recession

Series	Median 6 month effect
GDP Growth	-2.19
Inflation Change	-0.45
Policy Rate Change	-1.30
LR Interest Rate Change	-0.51
Stock Index Change	-1.5%

[9]All other data series in the X^{World} vector are assumed to be unknown.

Table 13 reports the estimates and ranges of world sector structural shocks to create the scenario described in Table 12. The most likely combination of shocks to push the U.S. economy away from a global structural steady state include a large negative demand shock, a modest negative shock to global productivity, a marginal negative price shock and a significant monetary policy shock that lowers interest rates.

Table 13: Estimates of Standardized World Sector Shocks from U.S. Recession Inputs

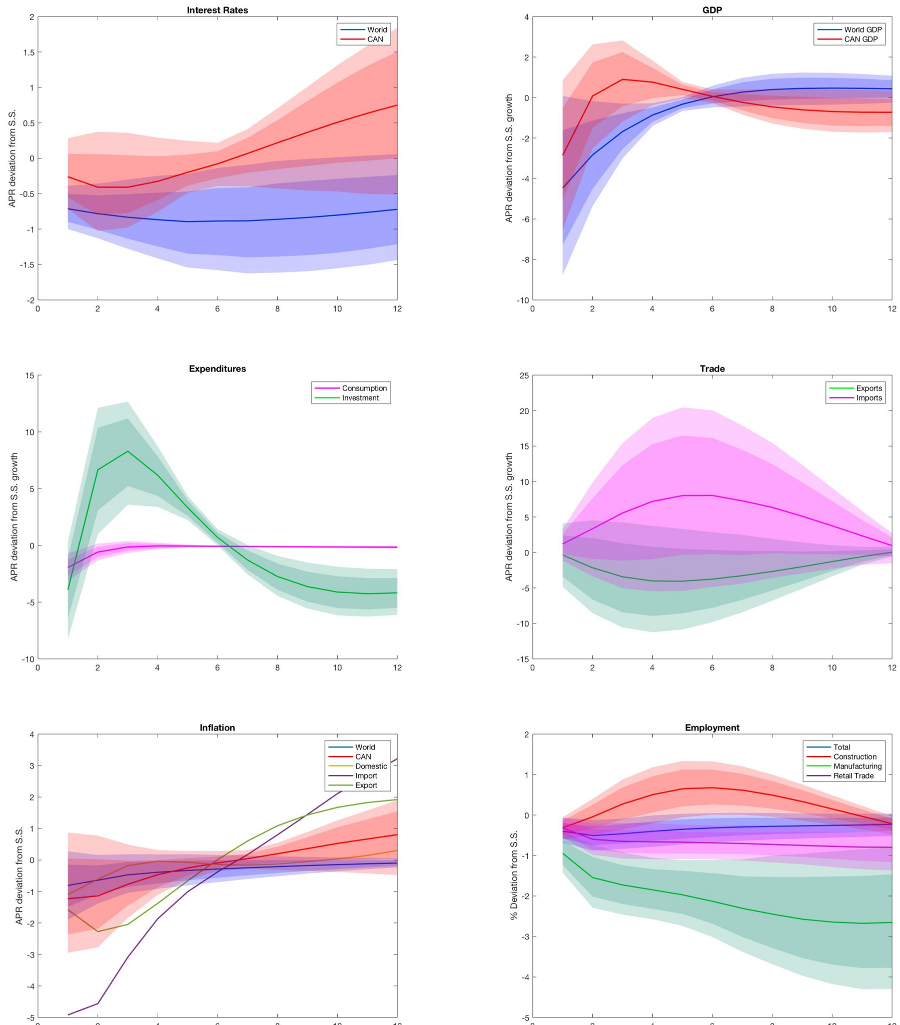
	Shock	Median	5%	95%
ε_{R^*}	World Interest Rate shock	-2.1	-3.2	-1.1
μ_{π^*}	World Price shock	-0.6	-1.9	0.5
ζ	Global Productivity shock	-1.1	-1.8	-0.3
z_{y^*}	World Demand shock	-3.0	-6.4	-0.2

Figure 8 plots the economic response of the world economy and the Canadian economy to such a combination of shocks. The dark lines plot the median IRFs from the combination of shocks while the lighter shaded regions plot the one and two standard deviations of the IRFs. The posterior density band take into account two types of uncertainties by sampling the DSGE-DFM posterior estimates (parameter uncertainty) and sampling the different combinations of world sector shocks that “created” the U.S. recession (world sector state uncertainty from kalman filter).

A U.S. recession calibrated to its post WWII averages for GDP growth, inflation growth, interest rate change and stock market growth declines would probabilistically result in negative Canadian GDP growth and World GDP growth, with the world declining by a larger amount than Canada does. The policy rate for Canada and global central banks would both fall. World interest rates would fall in the resulting first year, with Canadian rates falling by 50 basis points while global rates would fall by 100 basis points on average. The Canadian policy rate would recover by about 6 quarters after the shocks, while global interest rates would remain below average for the medium term.

Consumption would first fall in Canada but recover quickly, while investment would first fall as well, but significantly recover, due to the lower interest rate and inflation environment. The interest rate differential between Canada and the world, strengthens the Canadian dollar

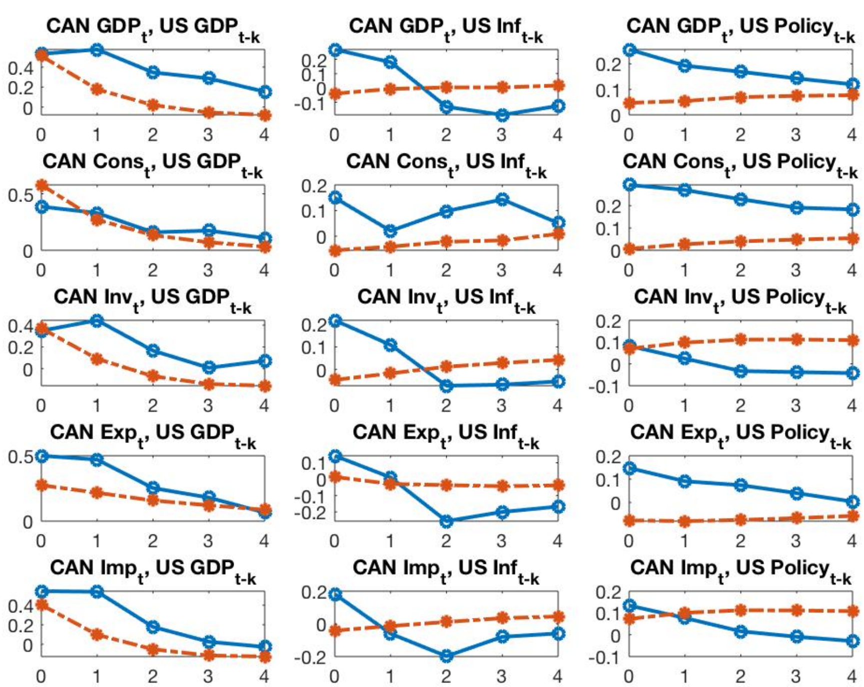
Figure 8: CAN Response to Average U.S. Recession



and stimulates imports and shrinks exports a few quarters after the initial combination of shocks. The resulting decline of exports and increase in investment and imports leads to an eventual increase in import inflation. It also has heterogenous effects in the Canadian labor market. Overall employment and retail employment falls slightly due to the decline in Canadian output. After initially falling, employment in the construction sector rebounds and grows above trend after investment begins to rise from the interest rate decline. Finally, employment in the manufacturing sector falls over the next 3 years after the combination of shocks, as export growth significantly declines and export inflation eventually increases.

Figures 9 give the cross-correlations between Canadian data and lagged U.S. data as well as the cross-correlations seen between the two countries in the simulated DSGE-DFM model for various expenditure series. When I evaluate the model implied correlation between the Canadian economy and the U.S. economy based on 10000 simulations, I find that GDP growth and its components do resemble those seen in the actual data. For example, the contemporaneous cross-correlations between Canadian GDP, consumption, investment, export and import growth with U.S. GDP growth seen in the sample data are: 0.51, 0.39, 0.35, 0.5 and 0.54 respectively. While the contemporaneous cross-correlations between the Canadian series and U.S. GDP growth in the simulated DSGE-DFM model are: 0.51, 0.58, 0.35, 0.28 and 0.40 respectively. The cross-correlations between these five Canadian series and U.S. inflation and the U.S. policy rate are all near zero for the simulated DSGE-DFM model. However, this is line with those seen in the data as they are all below 0.25 and most below 0.15.

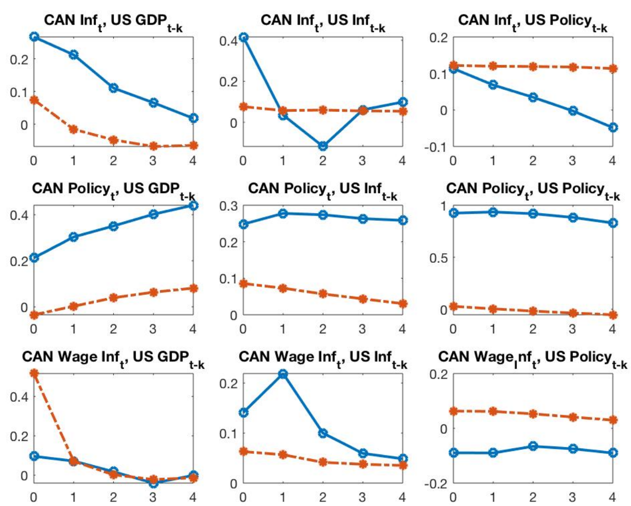
Figure 9: Data and DSGE-DFM cross-correlations for Canada and U.S.



Note: The solid blue line reports the sample correlation found in the data, while the dashed red line reports the correlation found in the simulated DSGE-DFM model. The x-axis represent the k lags of U.S. variables.

However, the simulated DSGE-DFM model is still not able to produce any policy rate correlation between the United States and Canada and a smaller than empirically observed cross-correlation for inflation between the two countries. This can be seen in the plots of Figure 10. The contemporaneous cross-correlation between Canadian inflation and the U.S. inflation is 0.42 in the data and only 0.11 in the simulated model. Further, the contemporaneous cross-correlation between the policy rates of Canada and the U.S. is 0.92 in the data and only 0.03 in the simulated model.

Figure 10: Data and DSGE-DFM cross-correlations for Canada and U.S.



Note: The solid blue line reports the sample correlation found in the data, while the dashed red line reports the correlation found in the simulated DSGE-DFM model. The x-axis represent the k lags of U.S. variables.

In summary, the model on its own exhibits empirically verified cross-correlations between U.S. output and Canadian output and its components but not for inflation or the policy rate. However, these co-movements for inflation and the policy rate between the two countries can exist when other U.S. series are exogenously calibrated as we saw in Figure 8.

8 Conclusion

In this paper, an open-economy New Keynesian dynamic stochastic general equilibrium (DSGE) model similar to the Lubik and Schorfheide et al. (2007) and Adolfson et al. (2007)

models is estimated using a large set of Canadian and international economic and financial series following the work of Boivin and Giannoni (2006) and Gelfer (2019). I denote a model estimated in this way as a DSGE-DFM model. I then conduct similar exercises to Ca'Zorzi et al. (2017), Kolasa and Rubaszek (2018) and Champagne, Poulin-Bellisle and Sekkel (2018), comparing the root mean squared errors (RMSE's) of the open-economy DSGE-DFM model to the RMSE's of the open-economy DSGE-Reg model and other forecasting models. I find that the DSGE-DFM model is better in capturing the dynamics of many economic series including output, investment, consumption, exports, imports and exchange rates around the time of the Great Recession and its ensuing recovery than is the open-economy DSGE model estimated in a traditional sense or other reduced-from forecasting models.

Further, the open-economy DSGE-DFM model that uses real-time data to generate its forecasts produces significantly better forecasts of output, consumption, investment, exports, imports and exchange rates when compared to the Bank of Canada's Staff Economic Projections (SEP). In addition, the DSGE-DFM model of this paper that incorporates real-time data significantly out-forecasts a random walk and all other reduced-form models in regard to the short-term and medium-term path of both the real and nominal exchange rate. An accomplishment that other open-economy DSGE models do not posses (Ca'Zorzi et al. 2017).

In addition, the Canadian DSGE-DFM model in this paper also sheds light on what can be expected to happen to the Canadian macroeconomy if the United States enters a recession. In summary, a U.S. recession calibrated to its post WWII averages for GDP growth, inflation growth, interest rate change and asset price declines would probabilistically result in negative Canadian GDP growth, and negative employment growth, with employment in the manufacturing sector declining the most. I see that the open-economy DGSE-DFM model does exhibit empirically driven correlation between the United States and Canada mending the cross-country linkage issue found by Justiniano and Preston (20010) for output and its components but not for inflation and the policy rate.

The continuing advancements in computational programming and the ever growing number of international macroeconomic and financial series available allows DSGE-DFM estimation to be a bountiful area of future research and model building.

References

Adolfson, M., Laséen, S., Lindé, J. and Villani, M. (2007), 'Bayesian Estimation of an open-economy DSGE Model with incomplete pass-through', *Journal of International Economics* **72**(2), 481–511.

Adolfson, M., Laséen, S., Lindé, J. and Villani, M. (2008), 'Evaluating an estimated New Keynesian small open-economy model', *Journal of Economic Dynamics and Control* **32**(8), 2690–2721.

Amirault, D., Wilkinson, G. L. and Kwan, C. C. (2006), *Survey of price-setting behaviour of canadian companies*, Bank of Canada.

An, S. and Schorfheide, F. (2007), 'Bayesian Analysis of DSGE Models', *Econometric Reviews* **26**(2-4), 113–172.

Boivin, J. and Giannoni, M. (2006), DSGE Models in a Data-Rich Environment, NBER Technical Working Papers 0332, National Bureau of Economic Research, Inc.

Cai, M., Del Negro, M., Giannoni, M. P., Gupta, A., Li, P. and Moszkowski, E. (2019), 'DSGE forecasts of the lost recovery', *International Journal of Forecasting* **35**(4), 1770–1789.

Calvo, G. A. (1983), 'Staggered prices in a utility-maximizing framework', *Journal of monetary Economics* **12**(3), 383–398.

Ca'Zorzi, M., Kolasa, M. and Rubaszek, M. (2017), 'Exchange rate forecasting with DSGE models', *Journal of International Economics* **107**, 127–146.

Champagne, J., Bellisle, G.-P. and Sekkel, R. (2018), Evaluating the Bank of Canada staff economic projections using a new database of real-time data and forecasts, Technical report, Bank of Canada Staff Working Paper.

Christoffel, K. P., Coenen, G. and Warne, A. (2010), 'Forecasting with DSGE models', *ECB working paper* .

Del Negro, M., Eusepi, S., Giannoni, M., Sbordone, A., Tambalotti, A., Cocci, M., Hasegawa, R. and Linder, M. H. (2013), The FRBNY DSGE model, Staff Reports 647, Federal Reserve Bank of New York.

Del Negro, M. and Schorfheide, F. (2013), DSGE Model-based Forecasting, *in* 'Handbook of economic forecasting', Vol. 2, Elsevier, pp. 57–140.

Dorich, J., Johnston, M., Mendes, R., Murchison, S., Zhang, Y. et al. (2013), *ToTEM II: An updated version of the Bank of Canada's quarterly projection model*, Bank of Canada.

Edge, R. M. and Gurkaynak, R. S. (2010), 'How Useful Are Estimated DSGE Model Forecasts for Central Bankers?', *Brookings Papers on Economic Activity* **41**(2), 209–259.

Gelfer, S. (2019), 'Data-rich DSGE Model forecasts of the great recession and its recovery', *Review of Economic Dynamics* **32**, 18–41.

Gervais, O., Gosselin, M.-A. et al. (2014), *Analyzing and forecasting the Canadian economy through the LENS model*, Bank of Canada.

Gurkaynak, R. S., Kisacikoglu, B. and Rossi, B. (2013), 'Do DSGE models forecast more accurately out-of-sample than VAR models?', *CEPR Discussion Paper No. DP9576* .

Holden, T. and Paetz, M. (2012), Efficient Simulation of DSGE Models with Inequality Constraints, School of Economics Discussion Papers 1612, School of Economics, University of Surrey.

Justiniano, A. and Preston, B. (2010), 'Can structural small open-economy models account for the influence of foreign disturbances?', *Journal of International Economics* **81**(1), 61–74.

Justiniano, A., Primiceri, G. E. and Tambalotti, A. (2011), 'Investment shocks and the relative price of investment', *Review of Economic Dynamics* **14**(1), 102–121.

Kolasa, M. and Rubaszek, M. (2018), 'Does the foreign sector help forecast domestic variables in DSGE models?', *International Journal of Forecasting* **34**(4), 809–821.

Kolasa, M., Rubaszek, M. and Skrzypczyński, P. (2012), 'Putting the New Keynesian DSGE Model to the real-time forecasting test', *Journal of Money, Credit and Banking* **44**(7), 1301–1324.

Lubik, T. A. and Schorfheide, F. (2007), 'Do central banks respond to exchange rate movements? A structural investigation', *Journal of Monetary Economics* **54**(4), 1069–1087.

Sims, C. A. (2002), 'Solving Linear Rational Expectations Models', *Computational Economics* **20**(1-2), 1–20.

Smets, F. and Wouters, R. (2003), 'An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area', *Journal of the European Economic Association* **1**(5), 1123–1175.

Smets, F. and Wouters, R. (2007), 'Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach', *American Economic Review* **97**(3), 586–606.

Stock, J. H. and Watson, M. W. (2011), 'Dynamic Factor Models', *Oxford Handbook of Economic Forecasting* **1**, 35–39.

Wolters, M. H. (2015), 'Evaluating point and density forecasts of dsge models', *Journal of Applied Econometrics* **30**(1), 74–96.

A Appendix: Measurement Equations and Data

The following list the measurement equations for the DSGE-Reg model.

$$World\ GDP\ Growth = 400(y^*_t - y^*_{t-1} + \zeta_t) \tag{A.1}$$
$$World\ Inflation = 400(\pi^*_t) \tag{A.2}$$
$$World\ Policy\ Rate = 400(R^*_t) \tag{A.3}$$
$$CAN\ GDP\ Growth = 400(y_t - y_{t-1} + \zeta_t) \tag{A.4}$$
$$CAN\ Consumption\ Growth = 400(c_t - c_{t-1} + \zeta_t) + \epsilon_{c,t} \tag{A.5}$$
$$CAN\ Investment\ Growth = 400(c_t - c_{t-1} + \zeta_t) + \epsilon_{i,t} \tag{A.6}$$
$$CAN\ Government\ Growth = 400(z_{G_t} - z_{G,t-1} + \zeta_t) + \epsilon_{g,t} \tag{A.7}$$
$$CAN\ Export\ Growth = 400(ex_t - ex_{t-1} + \zeta_t) + \epsilon_{ex,t} \tag{A.8}$$
$$CAN\ Import\ Growth = 400(im_t - im_{t-1} + \zeta_t) + \epsilon_{im,t} \tag{A.9}$$
$$CAN\ Hours\ Growth = 400(L_t - L_{t-1}) + \epsilon_{L,t} \tag{A.10}$$
$$CAN\ Wage\ Inflation = 400(\pi_{w,t}) \tag{A.11}$$
$$CAN\ Export\ Inflation = 400(\pi_{H^*,t}) \tag{A.12}$$
$$CAN\ Import\ Inflation = 400(\pi_{F,t}) \tag{A.13}$$
$$CAN\ CPI\ Inflation = 400(\pi_t) \tag{A.14}$$
$$CAN\ Real\ FX\ Growth = -100(s_t - s_{t-1}) \tag{A.15}$$
$$CAN\ Policy\ Rate = 400(R_t) \tag{A.16}$$

Table A.1: Data Series used in Estimation of the Canadian DSGE-Reg model

Data Set	Transform	Data Set	Transform
DSGE-Reg Estimation			
World Aggregate GDP Growth	0	CAN Import Growth	γ
World Aggregate Inflation	0	CAN Hours Growth	0
World Aggregate Policy Rate	β	CAN Wage Inflation	0
CAN GDP Growth	γ	CAN Export Inflation	0
CAN Consumption Growth	γ	CAN Import Inflation	0
CAN Investment Growth	γ	CAN CPI Inflation	0
CAN Government Growth	γ	CAN Real FX Rate Growth	0
CAN Export Growth	γ	CAN Policy Rate	β

Table A.2: Data Transformation Rubric

Code	Description
0	Demeaned
1	Linear detrended Log() per capita
2	Detrended Log()
γ	Calibrated steady state ζ is subtracted from growth rate
β	Calibrated steady state R is subtracted from policy rate

B Appendix: Tables and Figures

Table B.1: Calibrated Parameters

	Description	Value
World Sector		
β	Discount rate	0.9925
γ	World productivity growth rate	0.375
Resource Constraint		
C_y	S.S. Consumption proportion of CAN output	0.57
I_y	S.S. investment proportion of CAN output	0.19
G_y	S.S. government proportion of CAN output	0.24
α	Share of Imports of CAN output	0.31
δ	Depreciation rate	0.02
ψ_H	Share of capital in CAN production	0.3
ψ_F	Share of capital in World production	0.3
λ_w	Degree of wage markup	0.25
λ_F	Degree of import markup	0.25

Table B.2: Priors for DSGE Structural Parameters

	Description	Dist	Mean	Std		Description	Dist	Mean	Std
World Sector									
χ^*	World inflation indexation	Beta	0.3	0.1	$\rho_{\mu_{\pi^*}}$	AR(1) coef. on world price shock	Beta	0.5	0.2
κ^*	World Philips curve slope	Gam	2	0.8	$\rho_{z_{y^*}}$	AR(1) coef. on world demand shock	Beta	0.5	0.2
H^*	World habit persistence	Beta	0.5	0.15	ρ_ζ	AR(1) coef. on global prod growth	Beta	0.5	0.2
ϕ_{pi^*}	Taylor Rule coef. on World inflation	Norm	1.5	0.1	$\sigma_{\mu_{\pi^*}}$	Std. of world price shock	IG	0.5	0.4
ϕ_{y^*}	Taylor Rule coef. on World output	Norm	0.25	0.05	$\sigma_{z_{y^*}}$	Std. of world demand shock	IG	0.5	0.4
ϕ_{dy^*}	Taylor Rule coef. on World diff output	Norm	0.13	0.05	σ_ζ	Std. of global prod. growth shock	IG	0.5	0.4
ρ_r^*	Lagged coef. World Taylor Rule	Beta	0.75	0.1	σ_{R^*}	Std. of world monetary policy shock	IG	0.5	0.4
Domestic Economy									
Price Parameters									
γ_w	Wage inflation indexation	Beta	0.5	0.1	θ_w	Calvo wage stickiness	Beta	0.75	0.1
γ_H	Domestic inflation indexation	Beta	0.5	0.1	θ_H	Calvo domestic price stickiness	Beta	0.75	0.1
γ_{H^*}	Export inflation indexation	Beta	0.5	0.1	θ_{H^*}	Calvo export price stickiness	Beta	0.75	0.1
γ_F	Import inflation indexation	Beta	0.5	0.1	θ_F	Calvo import price stickiness	Beta	0.75	0.1
Structural Parameters									
h	Habit consumption	Beta	0.5	0.2	ψ	CRRA coef. on labor	Gam	1.5	0.3
λ_I	Investment adjustment cost	Norm	5	1	λ_u	Capital utilization costs	Beta	0.2	0.05
ϕ_B	Risk premium elasticity	IG	0.03	0.05	η	Elasticity of sub. of home goods	Gam	0.5	0.1
η_{mc}	World markup above marginal cost	Gam	1.5	0.3					
Taylor Rule Parameters									
ϕ_π	Taylor Rule coef. on inflation	Gam	1.5	0.25	ϕ_y	Taylor Rule coef. on output	Gam	0.25	0.05
ϕ_s	Taylor Rule coef. on FX growth	Norm	0	0.05	ρ_r	Taylor Rule coef. on lagged rate	Beta	0.75	0.1
AR(1) Coefficients									
ρ_{μ_w}	AR(1) coef. on wage shock	Beta	0.5	0.05	ρ_{μ_H}	AR(1) coef. on domestic price shock	Beta	0.5	0.05
$\rho_{\mu_{H^*}}$	AR(1) coef. on export price shock	Beta	0.5	0.05	ρ_{μ_F}	AR(1) coef. on import price shock	Beta	0.5	0.05
ρ_{z_H}	AR(1) coef. on domestic prod. shock	Beta	0.7	0.1	ρ_{z_D}	AR(1) coef. on export shock	Beta	0.7	0.1
ρ_{z_I}	AR(1) coef. on investment shock	Beta	0.7	0.1	ρ_{z_C}	AR(1) coef. on consumption shock	Beta	0.7	0.1
ρ_{z_G}	AR(1) coef. on gov't growth shock	Beta	0.7	0.1	ρ_{z_B}	AR(1) coef. on risk premium shock	Beta	0.7	0.1
Std. Coefficients									
σ_{μ_w}	Std. of wage shock	IG	0.5	0.4	σ_{μ_H}	Std. of domestic price shock	IG	0.5	0.4
$\sigma_{\mu_{H^*}}$	Std. of export price shock	IG	0.5	0.4	σ_{μ_F}	Std. of import price shock	IG	0.5	0.4
σ_{z_H}	Std. of domestic prod. shock	IG	0.5	0.4	σ_{z_D}	Std. of export shock	IG	0.5	0.4
σ_{z_I}	Std. of investment shock	IG	0.5	0.4	σ_{z_C}	Std. of consumption shock	IG	0.5	0.4
σ_{z_G}	Std. of gov't growth shock	IG	0.5	0.4	σ_{z_B}	Std. of risk premium shock	IG	0.5	0.4
σ_r	Std. of monetary policy shock	IG	0.15	0.1					

Table B.3: Posterior Estimates for DSGE-Reg Parameters

	Mean	5%	95%		Mean	5%	95%
World Sector							
χ^*	0.24	0.12	0.39	$\rho_{\mu_{\pi^*}}$	0.30	0.11	0.50
κ^*	1.52	0.79	2.51	$\rho_{z_{y^*}}$	0.85	0.76	0.92
H^*	0.66	0.54	0.77	ρ_ζ	0.85	0.63	0.95
ϕ_{pi^*}	1.43	1.26	1.60	$\sigma_{\mu_{\pi^*}}$	0.11	0.09	0.14
ϕ_{y^*}	0.28	0.21	0.34	$\sigma_{z_{y^*}}$	2.17	1.52	2.98
ϕ_{dy^*}	0.18	0.10	0.26	σ_ζ	0.17	0.09	0.31
ρ_r^*	0.89	0.87	0.91	σ_{R^*}	0.05	0.04	0.06
Domestic Economy							
Price Parameters							
γ_w	0.44	0.29	0.59	θ_w	0.38	0.30	0.45
γ_H	0.61	0.44	0.76	θ_H	0.63	0.57	0.68
γ_{H^*}	0.49	0.33	0.66	θ_{H^*}	0.62	0.58	0.66
γ_F	0.61	0.45	0.75	θ_F	0.38	0.32	0.43
Structural Parameters							
h	0.70	0.56	0.82	ψ	1.28	0.92	1.68
λ_I	1.11	0.67	1.89	λ_u	0.23	0.15	0.32
ϕ_B	0.02	0.01	0.03	η	0.39	0.37	0.41
η_{mc}	1.18	0.82	1.6				
Taylor Rule Parameters							
ϕ_π	1.53	1.38	1.70	ϕ_y	0.15	0.11	0.20
ϕ_s	-0.04	-0.09	0.01	ρ_r	0.89	0.87	0.92
AR(1) Coefficients							
ρ_{μ_w}	0.54	0.45	0.63	ρ_{μ_H}	0.64	0.57	0.70
$\rho_{\mu_{H^*}}$	0.62	0.56	0.68	ρ_{μ_F}	0.72	0.68	0.77
ρ_{z_H}	0.71	0.61	0.79	ρ_{z_D}	0.68	0.58	0.76
ρ_{z_I}	0.94	0.86	0.97	ρ_{z_C}	0.88	0.74	0.96
ρ_{z_G}	0.95	0.92	0.97	ρ_{z_B}	0.94	0.92	0.97
Std. Coefficients (x 100)							
σ_{μ_w}	4.54	3.15	6.16	σ_{μ_H}	5.16	4.24	6.30
$\sigma_{\mu_{H^*}}$	9.06	7.78	10.48	σ_{μ_F}	4.92	3.98	5.92
σ_{z_H}	1.17	1.03	1.33	σ_{z_D}	1.58	1.38	1.79
σ_{z_I}	1.80	1.06	3.43	σ_{z_C}	2.09	1.38	3.09
σ_{z_G}	0.81	0.71	0.94	σ_{z_B}	0.80	0.59	1.03
σ_r	0.10	0.09	0.12				

Table B.4: Posterior Estimates for DSGE-DFM Parameters

	Mean	5%	95%		Mean	5%	95%
World Sector							
χ^*	0.18	0.09	0.30	$\rho_{\mu_{\pi^*}}$	0.34	0.18	0.51
κ^*	0.65	0.30	1.12	$\rho_{z_{y^*}}$	0.89	0.86	0.93
H^*	0.76	0.73	0.80	ρ_{ζ}	0.21	0.11	0.33
ϕ_{pi^*}	1.52	1.37	1.69	$\sigma_{\mu_{\pi^*}}$	0.11	0.09	0.14
ϕ_{y^*}	0.10	0.05	0.15	$\sigma_{z_{y^*}}$	2.22	1.80	2.81
ϕ_{dy^*}	0.12	0.04	0.20	σ_{ζ}	0.99	0.88	1.11
ρ_r^*	0.90	0.88	0.92	σ_{R^*}	0.07	0.06	0.08
Domestic Economy							
Price Parameters							
γ_w	0.36	0.22	0.47	θ_w	0.20	0.17	0.22
γ_H	0.50	0.40	0.62	θ_H	0.26	0.21	0.31
γ_{H^*}	0.38	0.30	0.48	θ_{H^*}	0.86	0.82	0.89
γ_F	0.22	0.13	0.30	θ_F	0.39	0.35	0.42
Structural Parameters							
h	0.53	0.50	0.57	ψ	0.62	0.47	0.75
λ_I	0.11	0.08	0.15	λ_u	0.24	0.14	0.34
ϕ_B	0.01	0.01	0.02	η	0.59	0.53	0.64
η_{mc}	1.19	0.94	1.40				
Taylor Rule Parameters							
ϕ_π	1.40	1.32	1.46	ϕ_y	0.20	0.16	0.24
ϕ_s	-0.11	-0.17	-0.05	ρ_r	0.84	0.83	0.86
AR(1) Coefficients							
ρ_{μ_w}	0.56	0.459	0.61	ρ_{μ_H}	0.60	0.53	0.66
$\rho_{\mu_{H^*}}$	0.58	0.53	0.63	ρ_{μ_F}	0.58	0.49	0.65
ρ_{z_H}	0.90	0.87	0.93	ρ_{z_D}	0.64	0.57	0.69
ρ_{z_I}	0.96	0.95	0.97	ρ_{z_C}	0.85	0.74	0.92
ρ_{z_G}	0.91	0.86	0.95	ρ_{z_B}	0.96	0.93	0.97
Std. Coefficients (x 100)							
σ_{μ_w}	0.95	0.7	1.37	σ_{μ_H}	0.88	0.70	1.09
$\sigma_{\mu_{H^*}}$	0.73	0.23	2.26	σ_{μ_F}	3.68	3.58	3.75
σ_{z_H}	0.39	0.31	0.47	σ_{z_D}	0.93	0.78	1.12
σ_{z_I}	0.27	0.21	0.35	σ_{z_C}	0.59	0.43	0.76
σ_{z_G}	0.55	0.44	0.68	σ_{z_B}	0.31	0.26	0.37
σ_r	0.14	0.12	0.15				

Table B.5: RMSE for Bank of Canada's Staff Economic Projections (SEP)

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP	1.60	1.89	3.33	3.33	1.88	1.53	1.82	1.19
Consumption	3.51	3.62	2.09	1.60	1.79	2.10	1.36	1.02
Investment	15.80	16.26	16.44	16.08	13.23	10.29	8.97	6.59
Government	4.66	3.13	3.24	4.99	3.79	2.64	3.73	2.69
Exports	7.67	8.23	11.10	10.55	8.26	5.43	4.42	2.94
Imports	8.04	9.99	11.04	11.04	8.16	7.08	5.96	3.71
Policy Rate	0.07	0.36	1.05	1.61	1.91	0.52	1.66	1.07
CPI	0.89	2.09	2.19	1.95	1.65	0.93	0.88	0.52
Core CPI	0.44	0.62	0.71	0.70	0.65	0.42	0.50	0.35

Table B.6: DM test statistics for CAN Expenditure Growth

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP								
DSGE-Reg vs DSGE-DFM	**3.8**	**2.5**	1.4	**2.7**	**2.1**	**2.0**	**2.2**	1.4
DSGE-DFM vs DSGE-DFM-RT 2	**2.0**	0.9	0.5	-0.4	1.1	0.9	-0.0	1.1
DFM-RT 2 vs DSGE-DFM-RT 2	1.2	0.8	0.8	1.0	0.5	0.9	1.6	**2.0**
SEP vs DSGE-Reg	**-4.0**	**-2.3**	-1.4	0.2	**-2.5**	**-2.0**	-0.3	-1.4
SEP vs DSGE-DFM	**-2.6**	-1.7	0.1	1.3	-0.8	-1.5	1.0	-1.1
SEP vs DSGE-DFM-RT 2	-1.0	-1.7	0.4	1.6	1.4	**-2.0**	1.2	-0.1
Consumption								
DSGE-Reg vs DSGE-DFM	-0.6	0.8	**3.2**	**2.2**	**3.6**	**3.0**	**2.7**	**2.1**
DSGE-DFM vs DSGE-DFM-RT 2	1.2	1.0	-1.7	0.4	-0.9	1.5	0.0	0.8
DFM-RT 2 vs DSGE-DFM-RT 2	1.3	1.3	**2.1**	1.3	0.3	1.2	**4.3**	**2.1**
SEP vs DSGE-Reg	1.6	1.0	-1.2	-1.3	-0.8	0.6	-1.1	-0.9
SEP vs DSGE-DFM	1.6	1.1	1.1	1.6	**2.5**	1.3	1.4	1.5
SEP vs DSGE-DFM-RT 2	**2.0**	1.1	1.0	1.6	**2.6**	1.3	1.4	1.5
Investment								
DSGE-Reg vs DSGE-DFM	1.4	1.7	**2.0**	1.2	0.7	**2.7**	1.3	**3.7**
DSGE-DFM vs DSGE-DFM-RT 2	**2.7**	0.5	1.1	-0.6	0.8	1.2	-0.9	**2.8**
DFM-RT 2 vs DSGE-DFM-RT 2	1.7	0.2	1.1	1.3	0.9	0.9	1.4	1.3
SEP vs DSGE-Reg	-0.8	-0.9	-1.7	0.3	0.3	-0.9	0.5	-1.7
SEP vs DSGE-DFM	0.2	1.0	-0.1	1.0	**2.1**	0.6	1.4	1.1
SEP vs DSGE-DFM-RT 2	**2.3**	1.0	0.9	1.2	**2.4**	1.4	1.3	1.1
Government								
DSGE-Reg vs DSGE-DFM	1.5	0.6	0.6	-1.9	**-3.4**	0.9	**-4.1**	0.3
DSGE-DFM vs DSGE-DFM-RT 2	-0.3	1.0	**-3.4**	-0.0	**-2.3**	-1.3	-1.1	-0.9
DFM-RT 2 vs DSGE-DFM-RT 2	0.8	0.2	-1.2	-1.0	-1.0	-0.2	-1.0	-0.8
SEP vs DSGE-Reg	1.4	-0.3	0.0	1.7	1.3	0.0	1.3	0.8
SEP vs DSGE-DFM	1.9	-0.0	0.2	1.6	1.1	0.5	1.2	0.7
SEP vs DSGE-DFM-RT 2	**2.0**	0.0	0.1	1.6	1.0	0.4	1.2	0.7

Note: The table shows Diebold-Mariano test statistic. Negative test statistics imply the RMSE are lower for the model on the left. Values in bold are significant at the 95% level.

Table B.7: DM test statistic for CAN Trade Growth, CPI, and Policy Rate

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
Exports								
DSGE-Reg vs DSGE-DFM	1.4	1.1	0.2	-0.4	-1.7	1.3	-0.2	0.8
DSGE-DFM vs DSGE-DFM-RT 2	**2.0**	0.8	**3.6**	-1.5	0.1	1.1	-0.3	1.0
DFM-RT 2 vs DSGE-DFM-RT 2	1.6	**16.7**	1.4	**2.1**	1.4	1.0	**2.9**	**4.4**
SEP vs DSGE-Reg	-1.2	-1.2	-0.6	-0.8	1.5	-1.2	-1.6	-1.1
SEP vs DSGE-DFM	-0.9	-1.2	-0.6	-0.7	1.5	-1.1	-1.1	-1.0
SEP vs DSGE-DFM-RT 2	1.0	-0.0	-0.1	-0.1	1.5	-1.1	-1.1	-0.7
Imports								
DSGE-Reg vs DSGE-DFM	0.5	0.2	**8.2**	0.4	**6.0**	-0.2	1.1	1.5
DSGE-DFM vs DSGE-DFM-RT 2	**2.0**	0.8	**-2.4**	1.3	0.5	1.6	1.4	1.2
DFM-RT 2 vs DSGE-DFM-RT 2	1.7	1.0	1.1	1.4	-1.0	1.0	1.2	1.3
SEP vs DSGE-Reg	-1.0	-0.7	-1.2	0.5	-1.8	-1.2	0.0	-1.1
SEP vs DSGE-DFM	-0.9	-0.9	-0.8	1.9	0.7	-1.4	**2.1**	0.2
SEP vs DSGE-DFM-RT 2	1.1	0.8	-0.8	1.6	1.4	-0.3	**2.1**	0.6
Policy Rate								
DSGE-Reg vs DSGE-DFM	-1.4	-0.6	0.0	0.5	0.8	-0.3	0.7	0.3
DSGE-DFM vs DSGE-DFM-RT 2	1.7	1.5	1.5	1.3	1.4	1.4	1.4	1.3
DFM-RT 2 vs DSGE-DFM-RT 2	**3.1**	**2.0**	0.6	0.0	-0.2	1.5	-0.0	0.4
SEP vs DSGE-Reg	**-2.0**	**-2.7**	-1.4	-1.7	-1.5	**-2.4**	**-2.0**	-1.7
SEP vs DSGE-DFM	-1.7	-1.4	-1.2	-0.8	-0.6	-1.3	-0.9	-1.0
SEP vs DSGE-DFM-RT 2	-0.8	-1.0	-0.8	-0.4	-0.1	-1.0	-0.3	-0.5
CPI								
DSGE-Reg vs DSGE-DFM	1.4	1.5	1.0	1.2	1.1	1.2	1.4	0.9
DSGE-DFM vs DSGE-DFM-RT 2	1.5	-0.3	1.1	1.7	0.3	1.5	1.1	1.5
DFM-RT 2 vs DSGE-DFM-RT 2	**2.6**	-1.3	-1.2	-1.0	-0.9	-0.9	-1.1	-0.8
SEP vs DSGE-Reg	**-2.7**	**-3.1**	-1.5	**-2.0**	**-2.1**	**-2.2**	**-2.7**	-1.8
SEP vs DSGE-DFM	-1.4	-0.5	-0.8	-0.7	-0.7	-1.7	-1.6	-1.6
SEP vs DSGE-DFM-RT 2	-0.9	-0.5	-0.3	-0.3	-0.6	**-2.3**	-1.6	-1.6

Note: The table shows Diebold-Mariano test statistic. Negative test statistics imply the RMSE are lower for the model on the left. Values in bold are significant at the 95% level.

Table B.8: Relative RMSE for the real and nominal exchange rates

	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 6$	$\tau = 8$
Real Exchange Rate						
RW vs VAR(2)	-1.2	**-3.2**	**-2.9**	**-3.0**	-1.4	-1.4
RW vs DFM	-1.4	**-2.4**	**-8.0**	**-4.4**	-1.6	-1.5
RW vs DFM-RT 2	**2.3**	1.8	**2.5**	-0.8	**-4.6**	-1.9
RW vs DSGE-Reg	-0.6	0.1	0.4	0.6	0.7	0.8
RW vs DSGE-DFM	1.1	0.9	1.3	1.4	**2.5**	1.3
RW vs DSGE-DFM-RT 2	**2.2**	**2.0**	**2.1**	**4.1**	**2.2**	1.4
Nominal Exchange Rate						
RW vs VAR(2)	-1.2	**-3.4**	**-3.1**	**-2.8**	-1.5	-1.4
RW vs DFM	-1.6	**-2.4**	**-5.6**	**-3.8**	-1.7	-1.5
RW vs DFM-RT 2	**2.0**	1.8	1.7	-0.3	-1.9	-1.5
RW vs DSGE-Reg	-0.7	-0.1	0.1	0.3	0.4	0.6
RW vs DSGE-DFM	1.3	1.1	1.3	1.6	1.7	-0.1
RW vs DSGE-DFM-RT 2	**2.0**	**2.0**	**2.0**	**2.3**	**2.7**	1.0
RW vs SEP	**2.0**	**2.0**	1.6	0.5	-0.5	-0.4

Note: The table shows Diebold-Mariano test statistic. Negative test statistics imply the RMSE are lower for the RW model. Values in bold are significant at the 95% level.

Figure B.1: Wage Shock (μ_w)

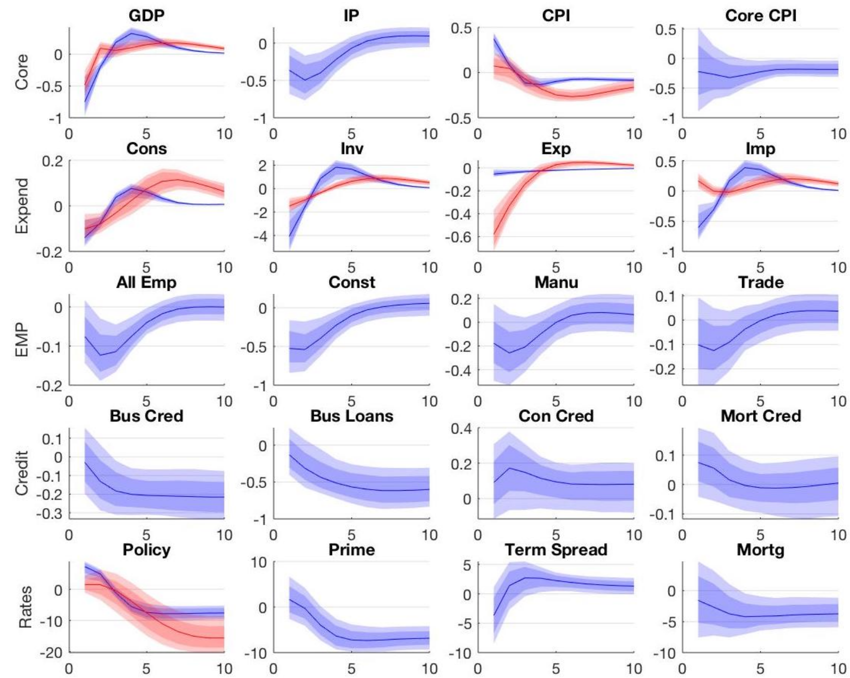


Figure B.2: Export Price Shock (μ_{H^*})

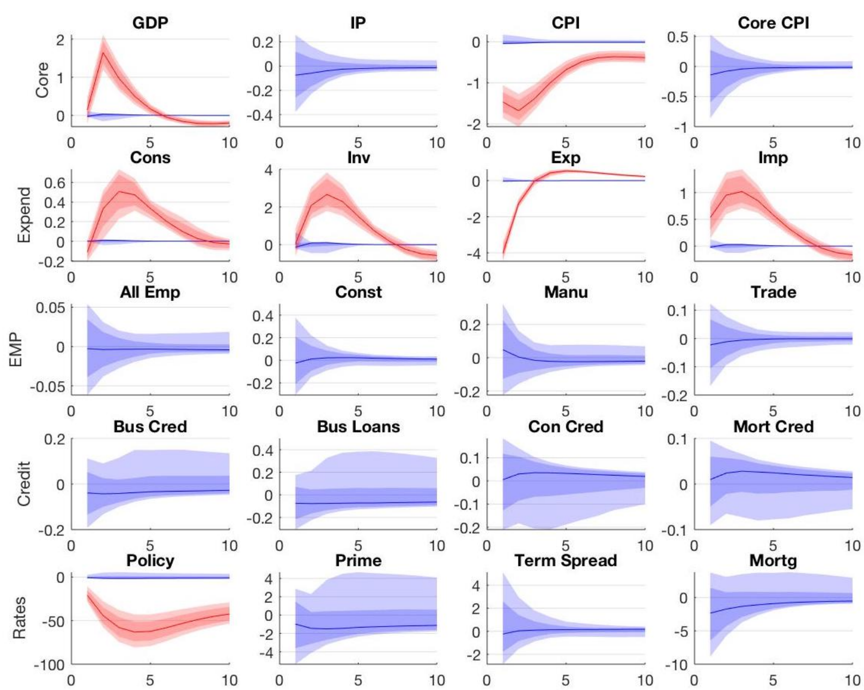


Figure B.3: Import Price Shock (μ_F)

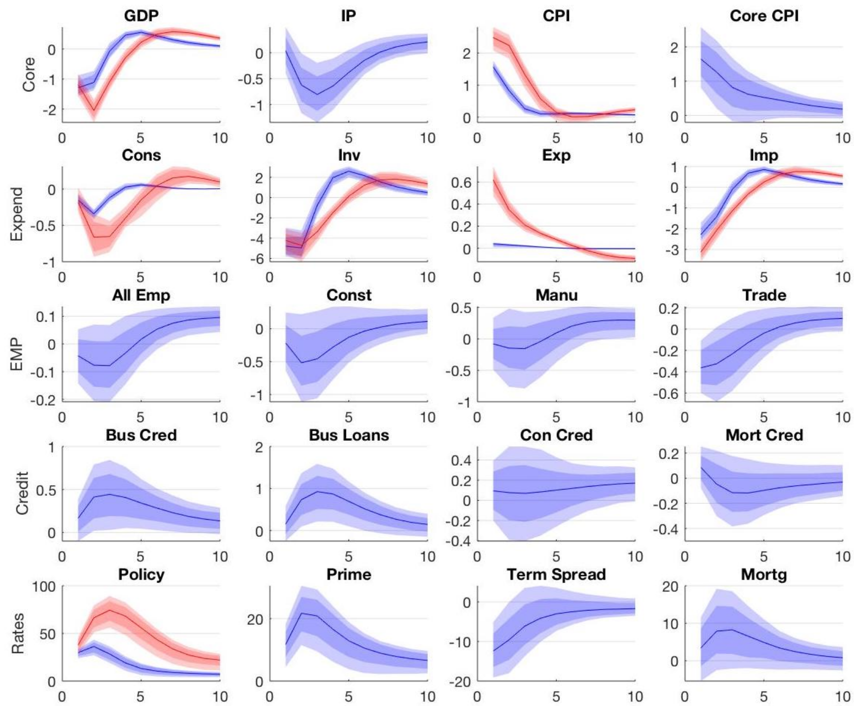


Figure B.4: World Price Shock (μ_{π^*})

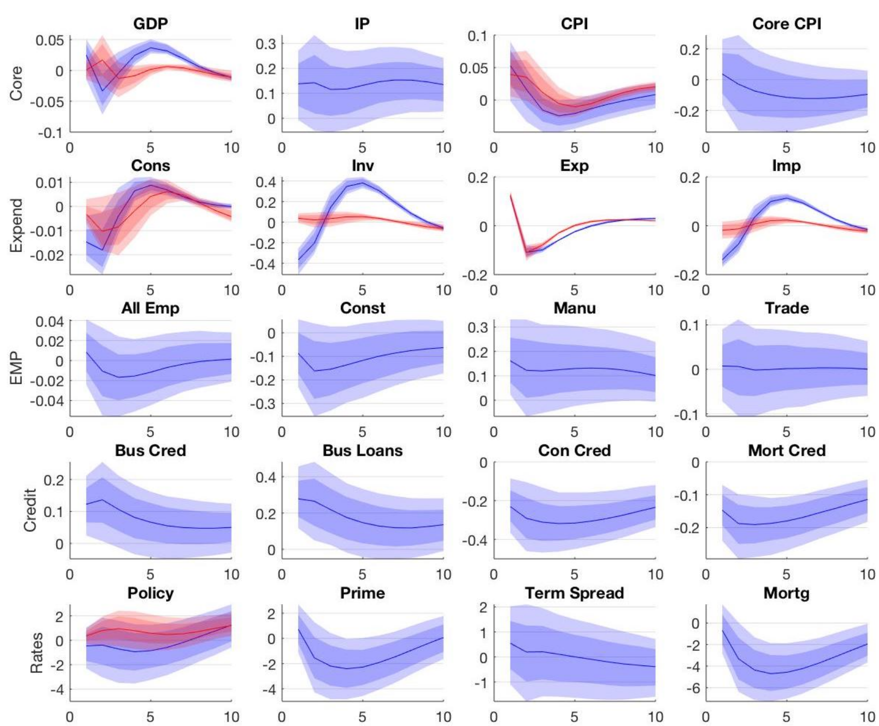


Figure B.5: World Interest Rate Shock (ε_{R^*})

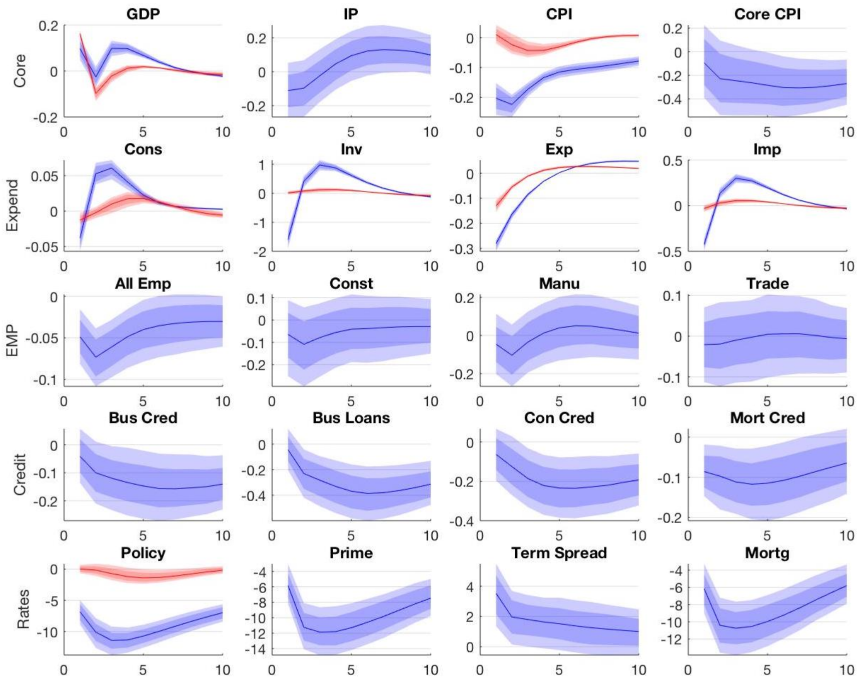


Figure B.6: Government Shock (z_G)

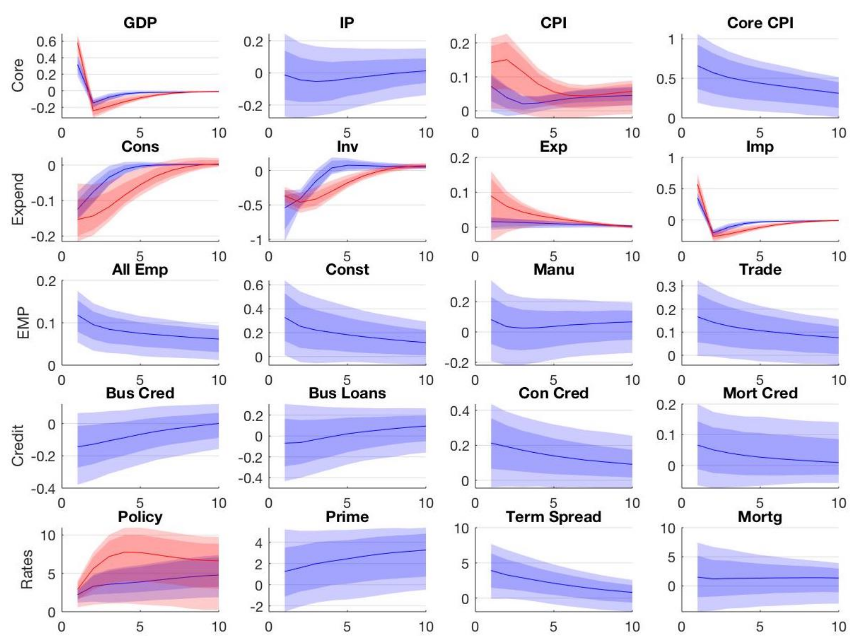


Figure B.7: Risk Premium Shock (z_B)

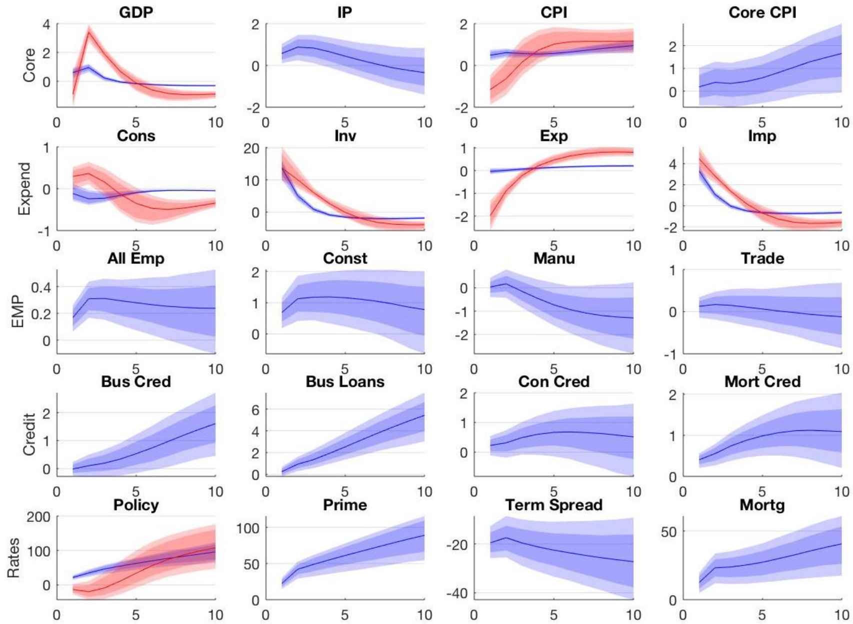


Figure B.8: Domestic Technology Shock (z_H)

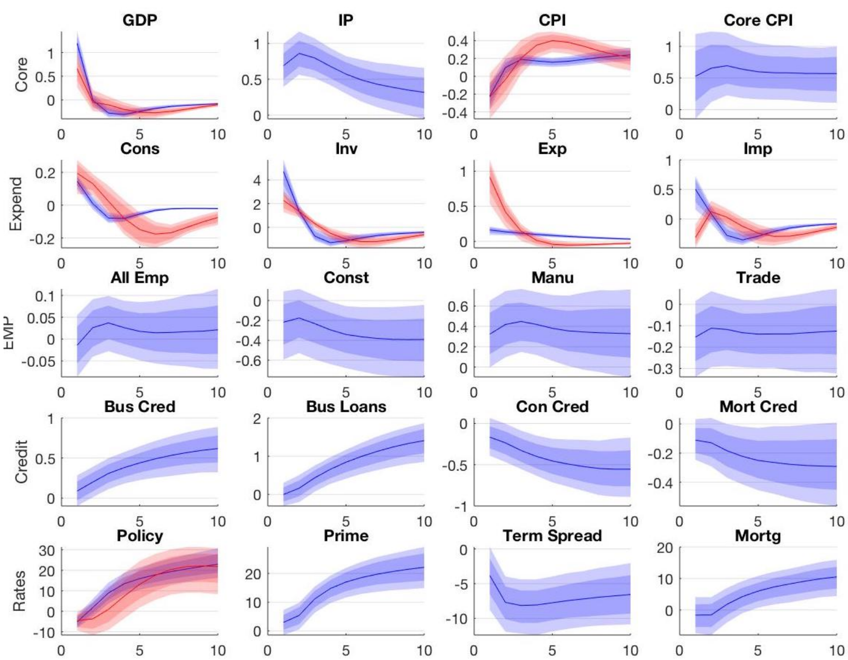


Figure B.9: World Sector Demand Shock (z_{y^*})

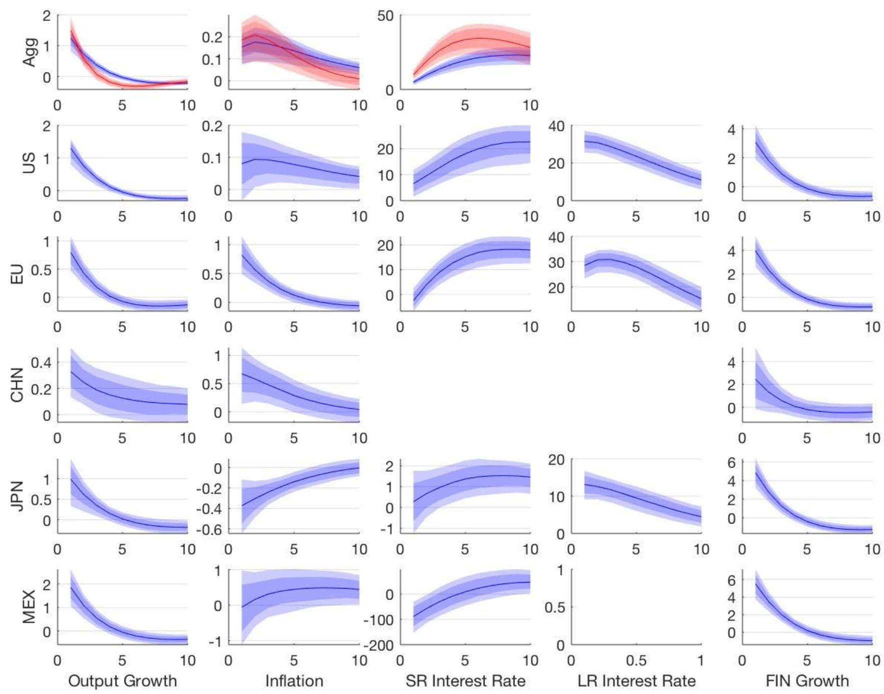


Figure B.10: World Sector Price Shock (μ_{π^*})

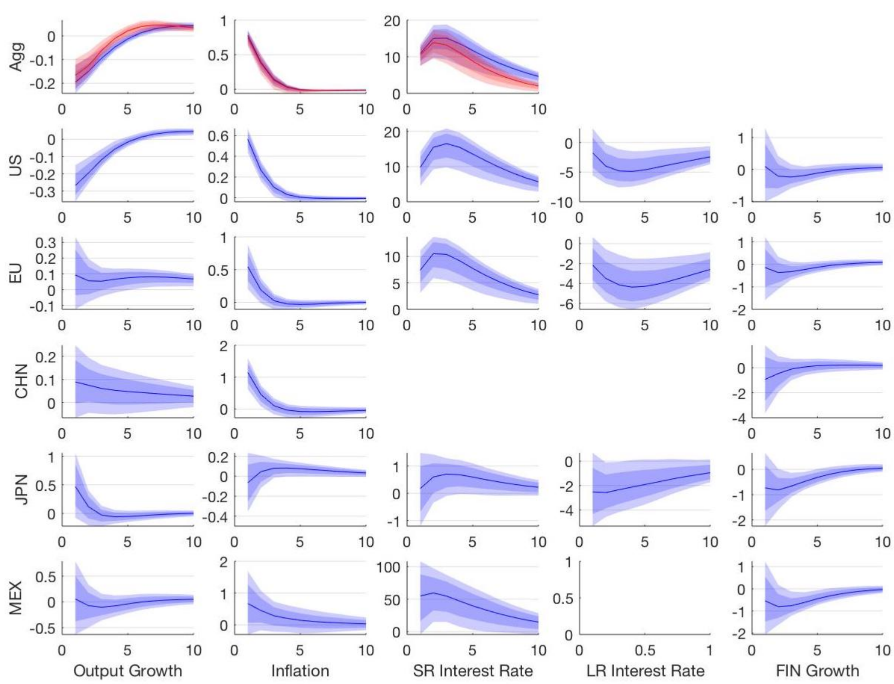


Figure B.11: World Sector Interest Rate Shock (ε_{R^*})

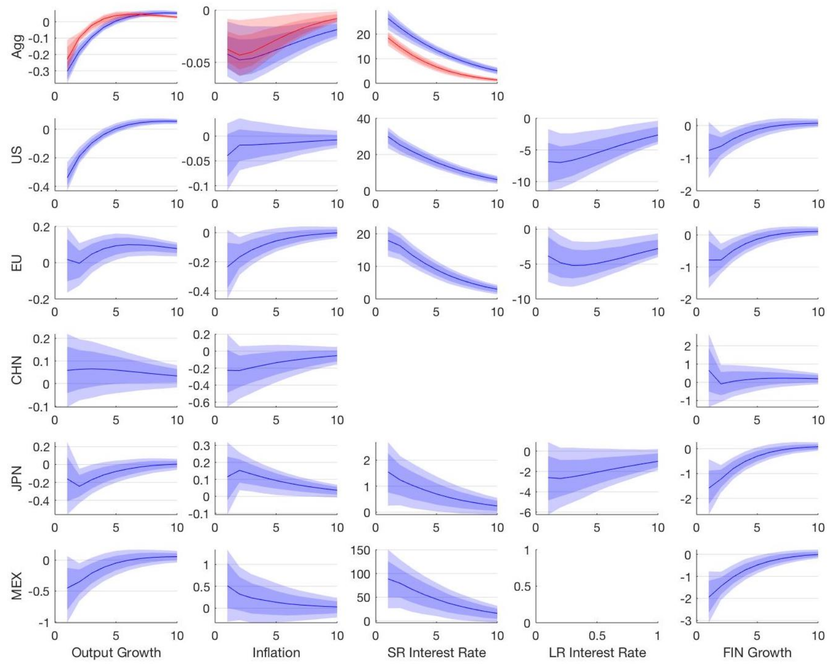


Figure B.12: World Sector Technology Shock (ζ)